AN ENTREPRENEURIAL DISCRETION MODEL: THEORY AND IMPLEMENTATION

By
FRANK SHERMAN MCLAUGHLIN, JR.

A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA
August, 1967

ACKNOWLEDGMENTS

The author would have never been in a position to begin, or complete this document if he had not received assistance, advice, and help from many people. Unfortunately, it is impossible to list, in this short space, all of those whom the author would like to thank. It is only possible to say, that for all this kindness, the author is exceedingly grateful.

Special thanks should go to three groups of people who played an especially significant role in the preparation of this dissertation. The members of the author's supervisory committee, Dr. W. V. Wilmot, Jr. (Chairman), Dr. E. L. Jackson, Dr. R. L. Lassiter, and Dr. M. E. Thomas, gave invaluable assistance. Mr. James W. Sikes, President of Florida Tile Industries, Inc., and other members of management at Florida Tile provided resources, assistance, and information without which this document could not have been written. Mrs. Carolyn Lyons assisted the author in many ways, most particularly in the final preparation of this dissertation.

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SECTION I

INTRODUCTION

There are many ways of describing a business firm. Some descriptions are very simple, others extremely complex. The simplest portrayal of a business firm consists of three parts; an entrepreneur, an objective function, and a production function. There are three major parts of this study. The first part is primarily concerned with modifying the simple portrayal of the firm in order to obtain a more realistic and workable model. The second part provides the background into which the portrayal should be set. The last part is an implementing section and is presented to show that the developed portrayal is realistic and is workable.

Plan of This Study

The first part of this study reviews the simple portrayal of the firm, and, then, using a stepwise procedure, modifies this simple portrayal until a more realistic and more workable model is obtained. Emphasis is placed upon the entrepreneur and the entrepreneurial process. The portrayal, therefore, is a management portrayal.

A major deficiency in the simple three-part portrayal is that it contains a single objective which the

firm strives to optimize. Most firms have many objectives. This study follows the thoughts of Simon (1959) that most objectives are "satisficing" rather than "optimizing" objectives. Furthermore, there seems to be a relationship between corporate objectives and the hierarchy of needs which serve as motivators of human action. Section III is offered to illustrate how the simple firm portrayal can be modified to incorporate these multiple objectives of the firm.

The simple portrayal of the firm is often criticized for its assumptions about the markets in which the firm operates, and for its assumption about the prices the firm pays for input factors and the prices the firm receives for its outputs. Section IV questions these assumptions, and attempts to determine which assumptions are sufficiently realistic to be included in a workable portrayal, and which assumptions must be modified. The necessary modifications are made and are incorporated into the developing portrayal.

As noted earlier, emphasis is placed upon developing a management portrayal of the firm. The simple portrayal includes an "omniscient power" which manipulates the variables in the production function and the objective function is an optimal manner. In order for the portrayal to be more correct, a more realistic concept of an entrepreneur must be presented. Section VI presents a concept of the

entrepreneur and the entrepreneurial process which is more realistic and workable, and which fits well into the developing portrayal of the firm.

The production function is the last part of the simple portrayal of the firm to be modified. Section VII redefines the production function, giving consideration to the entrepreneurial process presented in Section VI, the multiple objectives considered in Section III, and the marketing considerations of Section IV. This modified concept of a production function is more realistic than the concept of a production function used in the simple portrayal of the firm. This modified concept also fits well into the developing portrayal of the firm.

The next four sections are concerned with the background into which the portrayal should be set. Section VIII emphasizes the relationship between mathematical programming and the modified picture of the firm. Section IX and Section X show how the portrayal can be set into a modified accounting framework. These sections also show the compatibility of the modified accounting framework, the modified portrayal of the firm, and mathematical programming. Section XI is concerned with a partial theoretical formulation of the portrayal in terms of the concepts previously presented.

Although this dissertation is primarily concerned with the conceptual presentation, it was thought that significance would be added to the presentation if an implementing study was made. The remaining part of this study is devoted to this implementing study. Section XII describes the firm using the concepts presented in the study. Section XIII compares an actual and an optimal entrepreneurial solution. Section XIV is primarily devoted to review, summary, and general comments.

Basic Definitions and Assumptions

Before beginning the main work in this study, it would probably be wise to introduce some basic definitions and assumptions. Many definitions and many assumptions will be modified later. Attempts will also be made to justify some assumptions made here. For the moment, the reader is asked to accept the definitions and assumptions made. We must begin somewhere, and this is how we choose to begin.

Production may be defined as the process of combining and coordinating materials and resources in the creation of some valuable good or service. We do not limit the definition of production to the purely technical process of turning raw materials into finished products. Rather we accept such functions as accounting, sales and

personnel administration to be integral parts of the production process. Our definition is not so broad, however, as to include intangible items such as corporate image or community relations.

Any production process will consist of two parts, the *inputs* and the *outputs*. The output of a production process may be thought of as aggregates or sums of physical materials or resources. These materials and resources are the inputs or factor services of the production process. Thus production converts inputs into outputs. The term input and output must be envisioned with reference to the process, since a good or service which is an output from one production process may be the input to another. The inputs and outputs of the production process should be thought of as time flows of physical quantities. They may be hours of labor, kilowatts of electricity, or tons of fertilizer per year. The production process represents the transformation of input flows into ouput flows.

Economic theorists generally concern themselves with two types of productive services; those which vary with output and those which do not vary with the amount of output produced. These two types of services are generally referred to as *variable* and *fixed factors of production*. Raw materials and direct labor are often examples of

variable factors, while the services of a building may be thought of as a fixed factor.

The distinguishing characteristic between fixed and variable factors is not the technical feasibility of varying the factor, but rather it is the degree of variability of costs associated with utilizing this factor. Direct labor is normally thought of as being a variable cost; however if a guaranteed work week contract is in effect, direct labor may well be a fixed factor of production. To begin with, we shall assume that the firm possesses some factors of production which are fixed in quantity. The cost of these factors does not vary with the amount of output. We shall assume that the firm also utilizes some variable factors of production. The cost of these factors will vary with output.

In practice, a sharp dividing line seldom exists between fixed and variable productive services. Fixed services are generally only fixed within some limits of output variation. When output increases beyond these limits, some so-called fixed productive services may have to be increased, while some may remain fixed.

We shall define the term *business firm* to be a single productive unit. It may have many inputs and many outputs, but there must be some interrelationship between the inputs and the outputs. The firm is a total economic

unit under the control of just one "power." We shall call this "power" the *entrepreneur*. Thus the entrepreneur manages the business firm. The firm is the total economic unit over which the entrepreneur has financial control. It is the unit for which he calculates his profit and his loss. For our purposes, there is no need to distinguish between management and ownership. We can assume that the entrepreneur is both the manager and the owner.

Another assumption made is that the firm's production activity is arranged so that production in one time period is independent of the production in preceding and subsequent periods. The assumption is that the firm is interested in the activity of only one time period and that this activity is determined exclusively by conditions prevailing in that period and is independent of any other set of conditions. Following Carlson (1956), we shall refer to this as *mono-periodic production*. Mono-periodic production implies that production starts on a given date and ends at another date when the output is sold on the market. The time interval between the two dates represents the period under consideration.

In reality, it is impossible for a firm to arrange its operations so that production in one time period is independent of conditions in preceding and subsequent periods. The assumption of a mono-periodic production

period is, however, used extensively in economic literature to illustrate the optimizing procedures of a business firm. It should be remembered that such procedures are actually concerned with suboptimization rather than optimization, even though they are often referred to (as in this study) as optimizing techniques. A true optimizing procedure would have to consider conditions prevailing in the period under consideration as well as conditions in preceding and subsequent periods.

Demand conditions for final goods and services inform the entrepreneur about products that can be sold on the market. To the entrepreneur, demand appears as a series of possible price-quantity combinations which depend on prevailing market conditions and the firm's position in the market. To begin, we shall assume that the demand condition faced by the firm is essentially that of pure competition. The price a firm receives for any output has been predetermined by market forces. This price must be accepted by the firm as a predetermined parameter. It is not a variable which can be manipulated by the entrepreneur.

Pure competition implies that a firm may sell as much as it wants of any given product at the predetermined price. Later we shall modify this assumption. As we shall show later, the assumption of a predetermined price is not

a great departure from reality. The assumption that the firm may sell an infinite quantity is, however, a major departure from reality and must be modified later.

The *supply* of productive services, like the demand, will appear to the entrepreneur as a series of price-quantity combinations. For our purposes, however, we will again assume conditions which are essentially those of pure competition. The firm cannot influence the price by the quantity it takes. The factor prices are predetermined. Fixed factors of production are in the possession of the entrepreneur at the beginning of the mono-periodic production period. The cost of these factors is, in effect, sunk and cannot be varied.

The *technical knowledge* of the firm will be assumed to be fixed. Technical knowledge informs the entrepreneur of how a given output can be produced. Quite often there will be many (perhaps an infinite number) ways of combining inputs to obtain a desired output. It is the awareness of these different possible combinations which is known as technical knowledge or state of technology. It is this degree of awareness that is assumed not to change during the mono-periodic production period.

Finally, let us assume that the entrepreneur would like to maximize his total net return for the period under consideration. The validity of this assumption is highly

questionable. Most entrepreneurs however, seem to maneuver toward some goal. The assumption that they tend to optimize their profit position gives us a good place to start. It will be modified later.

In optimizing, the entrepreneur must concern himself with two broad types of problems. The first type is the purely technical problem of production. This type of problems pertains to the state of technology. It is concerned with the quantitative relation between inputs and outputs. The second type of problems are cost problems of production. These problems assume a given state of technical knowledge. They are concerned with the relation between the costs of different inputs and the value of different outputs. It is with this second type of problem that we will be primarily concerned in this study.

SECTION II

A SIMPLE PORTRAYAL OF THE FIRM: THE TRADITIONAL APPROACH

The simplest portrayal of a business firm consists of three parts: an entrepreneur, an objective function, and a production function. The entrepreneur is assumed to be the sole manager and proprietor of the business firm. The entrepreneur makes all decisions in the firm. Thus he manipulates the variables in the objective function and the production function in an attempt to achieve some objective. This objective is normally assumed to be profit maximization.

The Objective Function

The term objective function has made its way into economic and management literature primarily by the route of mathematical programming. The concept implies that a predetermined objective (for example, profit maximization) does exist. Furthermore, it assumes that the entrepreneur is fully aware of certain conditions such as demand for final products and supply of factor services. Unfortunately in problems involving more than a few aspects of the managing of a complex organization, it becomes most difficult to describe the objective function. As the complexity

of the organization grows, and as multiple goals are taken into account, the so called objective function becomes more and more subjective and less quantitative.

If we use the simplified conditions assumed in Section I, we can easily represent the firm's objective function mathematically,

$$P = \sum_{i=1}^{n} p_i x_i - \sum_{j=1}^{m} w_j y_j - S.$$

This is the firm's objective function--the function which the entrepreneur would like to maximize. It is also the firm's profit function, thus the implication that the firm would like to maximize profits. p_i is the constant price at which output i can be sold. w_j is the constant price at which input j can be bought. y_i represents the extent of utilization of input j and x_i represents the level or rate at which output i is produced. S represents the fixed or sunk costs which must be paid by the firm regardless of the rate of output.

The Production Function

We have assumed that a firm possesses a given amount of certain fixed factors of production. These factors, together with the state of technology, impose a set of technical relations which govern the possible transformation

of inputs into outputs. This relationship is the firm's production function.

The production function[1] may be conveniently expressed in mathematical form, writing output as a function of input,

$$F(x_1, x_2, \ldots, x_n) = G(y_1, y_2, \ldots, y_m).$$

The production function is always defined in relation to a given set of fixed factors of production. Furthermore the production function is defined to yield the maximum product obtainable from any specific combinations of input factors, given an existing state of technology.

It is best to visualize the production function as defining a number of constraints within which the firm must operate. As such, the production function is a boundary relationship which indicates the present limits of the firm's production possibilities. This relationship states that a firm cannot achieve a higher rate of output without using more inputs, and that fewer inputs cannot be used without decreasing the rate of output. The production function indicates the manner in which the firm can substitute inputs without varying the amount of output, and also

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[1]An alternate representation of the production function is given by

$$x_i = G_i(y_1, y_2, \ldots, y_m), \quad i = 1, \ldots, n.$$

the way in which the firm can substitute one output for another without altering its total usage of inputs. The production function represents only technically efficient operations by the firm. If a firm is not operating on its production function, then by shifting its operations to the production function, it can produce its present output with a smaller volume of inputs, it can use its present inputs to produce a larger volume of one or more outputs. If a firm wants to maximize profits, its operating possibilities are constrained to points on the production function.

The Entrepreneurial Solution

To complete our simple picture of the firm, it is only necessary to show how the entrepreneur will attempt to manipulate the variables under his control so as to obtain the maximum amount of profit which the constraints of the production function will allow.

Mathematically, the problem is easily solved using the Lagrange multiplier technique for solutions to constrained maximization problems. In practice, as we shall see later, the problem is never so easy. Nevertheless, we shall proceed with the mathematical solution for two reasons. First, it should provide the entrepreneur with an indication as to how to maximize. It should point him in the right direction. Second, we will be able to define

more clearly the relationship between this optimization problem and some concepts of traditional economic theory. The Lagrange function[2] may be expressed as

$$L = \sum_{i=1}^{n} p_i x_i - \sum_{j=1}^{m} w_j y_j - S + \lambda[F(x_1, \ldots, x_n) - G(y_1, \ldots, y_m)].$$

Invoking the necessary conditions for a maximum, we find,

$$\partial L/\partial x_i = p_i + \lambda(\partial F/\partial x_i) = 0, \quad i = 1, \ldots, n,$$
$$\partial L/\partial y_j = - w_j - \lambda(\partial G/\partial y_j) = 0, \; j = 1, \ldots, m,$$
$$\partial L/\partial \lambda = F(x_1, \ldots, x_n) = G(y_1, \ldots, y_m).$$

Rearranging the above equations, we obtain,

$$(\partial F/\partial x_1)/p_1 = \ldots, = (\partial F/\partial x_n)/p_n = \ldots,$$
$$= (\partial G/\partial y_1)/w_1 = \ldots, = (\partial G/\partial y_m)/w_m$$
$$= - 1/\lambda.$$

We have m + n + 1 independent equations in m + n + 1 unknowns. Theoretically, this system of equations can be solved for the values of the variables which maximize the firm's profit position. It is implicitly assumed that the maximum value of P is not less than -S. If P is less

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[2]The implicit assumption here is that the functions F and G are continuous and differentiable.

than -S, then the optimal solution[3] for the firm would be to shut down its operations, therefore making P = -S.

The mathematical solution to the stated problem is nice, but in its mathematical form it does little to describe what relationships the entrepreneur will strive to obtain in his quest for optimization. Before attempting to describe these relationships, it would probably be wise to introduce some concepts from economic theory.

Concepts from Economic Theory

The *law of diminishing returns* is a frequently quoted general economic principle. The law states that as equal increments of one input are added, while other inputs are held at a constant level, then beyond some point the resulting increments of output will decrease, i.e., the marginal product will diminish. The law holds equally well when a number of variable factors are increased in their most optimum proportions while other factors are held at a fixed level.

The law of diminishing returns assumes a given state of technical knowledge. It says nothing about the effect

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[3]It is important to point out here that S, the fixed or overhead costs of the firm, should only be used in determining whether the firm should operate or shut down. Beyond this, S plays no part in the production decisions of a firm which sells in a competitive market.

of adding units of any one input factor, holding the other factors constant, when the technological processes are also being changed. Also, there must be at least one fixed factor of production. The law of diminishing returns does not apply to a process in which all factors are variable. It also must be possible to vary the proportions in which the different input factors are combined. The law of diminishing returns is an empirical generalization. In most production processes which we can observe in the real world the law of diminishing returns seems to hold.

The law of diminishing returns implies the existence of diminishing returns in some parts of the production function. It does not, however, rule out the possibility of having increasing returns in other areas of the production function. When the ratio of variable factors to fixed factors is small, it is quite possible to be in a region of increasing marginal returns. As the proportion of variable factors is increased in relation to the fixed factors, we would expect to enter eventually a region of decreasing marginal returns. The proportion of variable factors could be increased to such an extent that total returns may actually diminish. It is obvious from the above discussion that the necessary conditions for a maximum are not also sufficient conditions. In order to insure that

a point obtained by the use of these equations is indeed a maximum, second order conditions must be investigated. Excellent discussions of second order conditions are given by Hancock (1960), Hadley (1964), and Gue and Thomas (to be published).

Marginal product, value of the marginal product, marginal cost, marginal rate of product transformation, and marginal rate of substitution are terms which also abound in economic literature. We shall define $\partial G/\partial y_j$, as the marginal product of input j. $\partial G/\partial y_j$ can be interpreted as an index of the marginal increase in total output resulting from a small increase in input j.

$\partial F/\partial x_i$ can best be interpreted as an index of the marginal increase in inputs required to produce a small increase of output i. From these definitions, it is easy to show that $-\lambda(\partial G/\partial y_j)$ is the value of the marginal product resulting from a change in any one input.

$$\text{TR(total revenue)} = \sum_{i=1}^{n} p_i x_i,$$

$$dTR = \sum_{i=1}^{n} p_i dx_i,$$

$$\sum_{i=1}^{n} (\partial F/\partial x_i) dx_i = \sum_{j=1}^{m} (\partial G/\partial y_j) dy_j,$$

$$dy_k = 0, \text{ if } k \neq j,$$

$$dy_j = (\partial G/\partial y_j)^{-1} \sum_{i=1}^{n} (\partial F/\partial x_i) dx_i,$$

$$dy_j = (\partial G/\partial y_j)^{-1} \sum_{i=1}^{n} - (p_i/\lambda)dx_i,$$

$$dTR/dy_j = [\sum_{i=1}^{n} p_i dx_i / \sum_{i=1}^{n} p_i dx_i](-\lambda)(\partial G/\partial y_j),$$

$$dTR/dy_j = - \lambda(\partial G/\partial y_j).$$

Likewise, $- \lambda(\partial F/\partial x_i)$ is the marginal cost resulting from a change in any one output. We shall define the marginal rate of substitution as the rate at which one input can be substituted for another while maintaining all outputs and all other inputs at a constant level. Thus, the marginal rate of substitution between inputs k and t may be defined mathematically as dy_k/dy_t. The marginal rate of product transformation is the rate at which one output may be substituted for another, while maintaining all inputs and all other outputs at a constant level. Using these definitions and the mathematical relationships previously derived, we can make several statements about the inputs to our optimal production process.

Optimizing Relationships in the Entrepreneurial Solution

The entrepreneur will attempt to operate so that the marginal product of the last unit of money spent of each input will be equal for every input,

$$(\partial G/\partial y_1)/w_1 = . . ., (\partial G/\partial y_j)w_j = . . .,$$
$$(\partial G/\partial y_m)/w_m.$$

The entrepreneur will attempt to utilize all inputs in such a manner that the value of their marginal product equals their market price,

$$- \lambda(\partial G/\partial y_j) = w_j, \qquad j = 1, \ldots, m.$$

The entrepreneur will attempt to operate in such a way that the marginal rate of substitution between every pair of inputs, holding all other inputs and outputs constant, is numerically equal to the inverse ratio of the input prices,

$$|dy_k/dy_t| = w_t/w_k, \qquad \text{all } k, t,$$

This is shown below.

$$\sum_{i=1}^{n} (\partial F/\partial x_i)dx_i = \sum_{j=1}^{m} (\partial G/\partial y_j)dy_j,$$

$$dy_j = 0, \quad j \neq k, t,$$

$$dx_i = 0, \quad i = 1, \ldots, n.$$

$$(\partial G/\partial y_k)dy_k + (\partial G/\partial y_t)dy_t = 0,$$

$$(\partial G/\partial y_t)/(\partial G/\partial y_k) = - dy_k/dy_t,$$

$$(\partial G/\partial y_t)/(\partial G/\partial y_k) = w_t/w_k,$$

$$|dy_k/dy_t| = w_t/w_k.$$

Following the same line of analysis as above, we can make the following statements about outputs in our optimal production process.

The entrepreneur will attempt to produce each output in such a manner that its selling price equals its marginal cost,

$$- \lambda(\partial F/\partial x_i) = p_i, \quad i = 1, \ldots, n.$$

The entrepreneur will attempt to operate in such a way that the rate of product transformation between every pair of outputs, holding all other outputs and inputs constant, is numerically equal to the inverse ratio of their prices,

$$|dy_k/dy_t| = p_t/p_k.$$

Again following the same line of analysis, we can make the following statements about the relationship between inputs and outputs.

In the optimal solution, the marginal value received from the last unit of input must be equal for every output,

$$p_1(\partial F/\partial x_i) = \ldots, p_i/(\partial F/\partial x_i)$$
$$= \ldots, p_n/(\partial F/\partial x_n).$$

In the optimal solution, the rate at which any input should be transformed into any output, holding all other inputs and outputs constant, is equal to the inverse ratio of their prices,

$$dx_i/dy_j = w_j/p_i, \qquad \text{all } i, j.$$

This is shown below.

$$(\partial G/\partial y_j)/(\partial F/\partial x_i) = w_j/p_i,$$
$$dx_i/dy_j = (\partial G/\partial y_j)/(\partial F/\partial x_i),$$
$$dx_i/dy_j = w_j/p_i.$$

In a few short pages, we have prescribed to the entrepreneur of our simple firm the procedure which he should follow in order to obtain the "most" from his scarce resources. The theory is fine; without it the entrepreneur would, perhaps, lack the direction in which to move. There are, however, two major weaknesses in what we have done. First we have oversimplified the picture of our firm. In fact, in our effort to simplify we have made some questionable assumptions about the firm's behavior. These will be discussed later. In the second place, theory alone is not enough. Tools must be developed (and existing tools must be utilized) to aid the entrepreneur in obtaining the most efficient utilization of his scarce resources.

There is, of course, nothing new about the preceding analysis. This is the type of analysis that classical economist have traditionally used to describe the firm. It has often been criticized because it tends to be more

of a narrative description rather than an implementing tool. In defense, it should be noted that the classical economists did not greatly concern themselves with the implementation of the theory they had developed. The economist accepted the production function as a description of the technological condition of production, and they accepted no direct responsibility for deriving it. The problem of efficiency was generally thought of as falling into the domain of the scientist or engineer.

A primary aim of economic theorists has been to understand business behavior rather than to make recommendations to business men. Economic theory, in effect, describes what a rational individual, who is well versed in decision-making, would do in his economic activities. The assumption of an optimal production function is important to the economist because it helps him understand the behavior of business men, consumers, and other members of the economy. Business knowledge and experience does allow buyers and sellers to arrive at decisions which come close to being optimal. Furthermore, competition often eliminates firms whose decision-making is consistently poor. Thus, the assumption of an optimal production function is somewhat valid, and to the extent that it is valid, economic theory serves as a relatively good description of economic behavior.

SECTION III

THE QUESTION OF MULTIPLE OBJECTIVES

Perhaps the most important criticism of our simple picture of the firm is that a typical firm does not possess just one objective function, but instead has many purposes and goals. In fact, if one goal could be selected as the paramount objective it is sometimes questioned if this goal would be profit maximization.

A Review of the Question

Baumol (1959) insists that consulting experience has shown him that firms attempt to maximize sales subject to a profit constraint. He believes that firms will attempt to sell as much (in terms of monetary revenue) as possible as long as a reasonable profit is made. Shubik (1961, p. 360) in a survey of twenty-five large corporations notes that such terms as "fair share, fair return, equitable wages, fair treatment, and proper return to investment" are often stated as company objectives.

Shubik (1961, p. 366) notes that a firm may often make a statement such as, "we wish to maximize profits and market share." It is quite possible that these two items may be negatively correlated through the effect of

independent variables. In this case, the cost of sales effort necessary to increase market shares may decrease profits. In some situations, however, the correlation between the factors which control the values of the stated goals of the firm may be sufficiently close to unity so that the maximization of one value maximizes the value of another (to a good approximation).

Peter Drucker (1954, p. 46) claims that "the guiding principle of business economics is not the maximization of profits; it is the avoidance of loss." To carry this reasoning one step further, survival of the firm must be considered as a paramount objective. The firm, as a social and economic organization, like may other organisms, has a compelling urge to survive. The motive to survive is probably more fundamental than the profit motive; it is implicit in most decisions within the firm. In the long run a firm that survives will make a positive profit. However, a firm that maximizes profits might not survive. Obviously, sufficiently large losses will bankrupt a firm. It is also possible to be a profitable firm, but because of inadequate liquidity, etc., not to survive. Thus the goal of survival may take precedence over all other objectives of the firm. In the short run, all positive profit may have to be sacrificed to permit survival.

Many entrepreneurs believe the key to survival lies in sound financial management. Financial management includes a variety of goals which can be generalized by stating that the financial requirements of all possible future conditions of the firm should be met adequately.

No firm can realize this objective completely; to prepare for one contingency adequately often limits a firm's ability to meet other situations. For instance, the relative amount of debt and equity financing should be determined on the basis of the worst possible earnings position which can reasonably be expected within the firm's economic horizon.

Some of the major types of financial limitations would be the maximum amount of short-term or long-term debt, the minimum ratio of current assets to fixed assets, the maximum amount of inventory or receivables in relation to sales, and the minimum level of working capital.

In addition to these basic relationships, financial management makes use of a wide variety of comparisons and ratios between the major components of the balance sheet and income statement. All these financial yardsticks can be applied as constraints in the solution of any nonfinancial problem of the firm.

Occasionally, in the short run, a financial objective may take precedence over all other objectives.

The liquidity position of the firm can be of prime importance and at times might be crucial. While cash is the only truly liquid asset, most firms regard receivables as liquid assets. The firm might want to maximize cash and receivables as a percentage of other total assets. Other financial ratios or relationships might be optimized in a similar manner.

The creation and maintenance of corporate images plays an important role in the modern firm. Several types of images are of significance. Ultimately, all decisions result from some sort of image in the mind of the decision-maker. However, this image is partially a reflection of what the decision-maker thinks is the image of his firm and its products that is held by various other groups. These groups include customers, employees, competitors, stockholders, suppliers, government officials, and the general public. Decisions result not from the actual images held by these groups but by what managers think these images are. The reaction of management to the images of various groups is not a passive one because it is generally recognized that a firm's actions can and do influence these images.

Depending upon the group involved, various types of images may be considered desirable by a firm. A desirable consumer image might include such aspects as

service, quality of products, fairness of price, leadership and innovation. The image held by competitors would involve fair dealing, efficiency, leadership in volume of sales, etc.

The entire problem of image creation is complicated by the fact that the existence of a characteristic does not necessarily create an image of it nor does an image insure the presence of the elements involved. Actual service may not create a consumer image of service. The image of economy in a product may best be created not by a low price per unit but rather by a package that appears to give more units for a given price.

The Relationship to Psychological Theories

Simon (1959) gives one of the best explanations of multiple objectives. To Simon, the critical assumption is that the firm does wish to maximize. Simon argues that the firm may not wish to maximize, but may simply want to earn a return that is regarded as satisfactory. In his analysis, Simon draws heavily from the field of psychology. He notes that while satiation plays no role in economic theory, it does enter rather predominantly into the treatment of motivation theory in psychology. In most psychological theories, the motive to act stems from drives, and action terminates when the drive is satisfied.

Moreover, the condition for satisfying a drive is not necessarily fixed, but may be specified by an aspiration level that itself may adjust upward and downward on the basis of experience.

To better understand this concept of multiple goal formulation, it is best to draw upon the field of psychology, rather than the field of economics. Business firms are run by men, therefore we would expect an analogy between corporate goals and human goals.

Maslow's hierarchy

A. H. Maslow (1954) lists five types of human needs which are arranged in a hierarchy from lower levels to higher levels. They are:

1. Physiological needs, such as hunger, thirst, and sex.

2. Safety needs, such as security, stability, and order.

3. Belongingness and love needs, such as needs for affection, affiliation, and identification.

4. Esteem needs, such as needs for prestige, success, and self respect.

5. Need for self-actualization.

The ordering of these needs is significant in two ways. It is the order in which they tend to appear in the normal development of the person and also the order in which they tend to be satisfied.

The individual attempts first to satisfy his physiological needs. Once these are relatively well satisfied, then higher order needs emerge and begin to dominate the individual. When these are in turn satisfied, again new (and still higher) needs emerge, and so on. This is what Maslow meant by saying that the basic human needs are organized into a hierarchy of relative prepotency. The individual is dominated and his behavior organized only by the unsatisfied needs. Thus, man lives by bread alone--when there is no bread. But man's desires change when there is plenty of bread and his stomach is chronically filled. A brief summary of the needs are given below.

Physiological needs.--Although it is impossible to make a complete list of physiological needs, certainly they would include hunger, thirst, sleep, sex, and body homeostasis.[1]

Undoubtedly these physiological needs are the most prepotent of all needs. To the human being who is missing everything in life in an extreme fashion, it is most likely that the major motivation would be physiological needs rather than any other. A person who is lacking in

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[1]Homeostasis refers to the body's automatic efforts to maintain a constant, normal state of the blood stream. Thus, if the body lacks some chemical, the individual will tend to develop a specific appetite or hunger for that food.

food, safety, love, and esteem would probably hunger for food more strongly than for anything else. If all the needs are unsatisfied, and the individual is dominated by the physiological needs, then all other needs may simply become non-existent or simply become pushed into the background. As Maslow (1954, p. 82) has said, "For the man who is extremely and dangerously hungry, no other interests exist but food. He dreams food, he remembers food, he thinks about food, he perceives only food, and he wants only food."

Safety needs.--If the physiological needs are relatively well satisfied, there then emerges a new set of needs, which are roughly characterized as safety needs. All that has been said of the physiological needs is equally true, although in a less degree, of these desires. The individual may equally well be wholly dominated by them. Most adults in our society are safe enough from wild animals, extremes of temperature, criminal assault, etc. Therefore, in a very real sense, man no longer has any safety needs as true motivators. We can, however, perceive the existence of safety needs in such phenomena as formation of labor unions, job tenure, and insurance. As we will note later, safety needs do seem to have some effect in corporate goal formulation.

Love needs.--If both the physiological and the safety needs are fairly well satisfied, there will emerge the love and affection needs, and the whole cycle will repeat itself. The person will now feel keenly about the absence of friends, or a wife, or children. He will hunger for affectionate relationships with people in general, or for a place in his group, and he will strive with great intensity to achieve this goal.

Esteem needs.--All people in our society have a need or desire for a stable, firmly based, usually high evaluation of themselves, for self-respect, self-esteem and for the esteem of others. These needs may be classified into two subsidiary sets. First, there are the desires for strength, achievement, adequacy, competence, confidence, freedom, and independence. Second, there are the desires for reputation, prestige, status, dominance, recognition, attention, appreciation, and importance.

Need for self-actualization.--Even if all the above needs are satisfied, discontent and restlessness will develop unless an individual is doing what he is fitted for. This need we may call self-actualization. It refers to man's desire for self-fulfillment, namely, to the tendency for him to become actualized in what he is potentially. The specific form that these needs will take will of course vary from individual to individual. The clear emergence of

these needs, however, usually rests upon prior satisfaction of the physiological, safety, love, and esteem needs.

The above discussion may have given the impression that these five sets of needs are arranged in such terms that when one need is satisfied, then another need emerges. This might lead to the false impression that a need must be fully satisfied before the next need emerges. In reality, most members of our society are partially satisfied and partially unsatisfied in all of their basic needs at the same time. A more realistic description of the hierarchy would be in terms of decreasing percentage of satisfaction as we go up the hierarchy. Perhaps an average member of society is satisfied 90% in his physiological needs, 75% in his safety needs, and 50% in his love needs, 35% in his self-esteem needs, and 15% in his self-actualization needs. The emergence of a new need is seldom a sudden phenomenon, but rather a gradual emergence. For example, if need A is satisfied only 10%, then need B may not be visible at all. However as need A becomes satisfied 40%, then need B may emerge 5%, etc.

One difficulty in the formulation of the need hierarchy is that needs are often not what they seem to be. A person who thinks he is hungry may actually be seeking more comfort or dependence. Conversely, it is possible to satisfy the hunger need in part by other

activities such as drinking water or smoking cigaretts. As another example, there are supposed to be people (Morgan, 1961) who seek self-esteem for the sake of love rather than for self-esteem itself. This sometimes apparent reversal of the hierarchy does not, however, diminish the value of the concept for our purpose.

Maslow's hierarchy and the firm's goals

It is easy to see that there is an analogy between Maslow's hierarchy and the multiple goals of the firm. The analogy, however, is difficult to formalize. Certainly there is a hierarchy of corporate goals, but the determination of the hierarchy is difficult.

Certainly the survival needs of the firm would be in the lowest possible set of the hierarchy. Unless, and until the firm can be sure of surviving it is unlikely that management will concern itself greatly with corporate images, etc. To paraphrase Maslow, a firm which is short of cash will probably dream cash, think about cash, perceive cash, and want only cash.

Maslow's safety needs are primarily evidenced in corporations as insurance contracts, engineering specifications, etc. It can be argued, however, that safety needs are responsible for the tendency toward consolidation, and for apparent price stability in certain industries. This latter concept will be considered again in the next section.

Corporate love needs and esteem needs vary, but typical needs are obvious to most people. Each individual could produce his own list of great length. Let us just say that, in general, these needs are human needs which are projected through the edifice of the business firm.

The business firm conception of self-actualization needs is again open to many interpretations. Maslow (1954) indicated that the need for self-actualization is a need for an individual to do what he can do best, and to do that in the best way he can. In business language, this is not far from the economist's concept of profit maximization.

The Effect of Satisficing Constraints

If we are to explain business behavior in terms of this psychological theory, we must expect the firm's goals not to be formulated in terms of maximization, but rather in terms of attaining a certain level or rate of profit, holding a certain share of the market or maintaining a certain level of sales. Firms would try to "satisfice" rather than to maximize.

If we use this theory, then it is obvious that we have removed the objective function from our picture of the firm. In its place we have added a series of additional constraints which the firm must satisfy. The objective

function could, of course, be thought of as being a function of the constraint equations.[2] Conceptually, however, it is more convenient to think of the firm as an organism which seeks a feasible solution (one that satisfies the constraints) to the problem with only lower order constraints attached. Once this feasible solution is attained, the firm will add the constraints from the next level of the hierarchy and search for a solution which satisfies all constraints. The process then will be repeated until all constraints are attached.

Let us assume that a firm has no objective function and that it is seeking a feasible solution with "all constraints attached." There are three possible results. First there might be a unique solution toward which the firm will proceed. The probability of this, however, is almost zero. Second, it is possible that the "satisficing" constraints placed upon the firm by itself are inconsistent. Thus there is no action the firm can take which will satisfy all constraints. The third possibility is that there will be a range or set of activities which will satisfy all constraints. With no objective function, it must be assumed that any combination of activities which

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[2] Some economists refer to this as the firm's utility function.

satisfies all of the constraint equations would be equally desirable. The last two possibilities are quite relevant to reality.

In the case of inconsistency, the firm has no feasible alternative. Its only course of action is to violate (drop or reevaluate) some of the constraints. In general, the firm will violate those constraints which are associated with goals in the higher hierarchy levels. It will continue to violate these constraints until a feasible solution is obtained.

The second possibility, that there is a range of activities which the firm considers as satisfactory is, in reality, ridiculous. Any entrepreneur will state that his firm is always trying to do better, always attempting to achieve optimality. Certainly we must place the objective function back into our picture. The question remains, what is the firm trying to optimize?

Shubik (1961, p. 368) handles the problem this way. He says, suppose a firm states that "it wishes to maximize profits, maintain growth, and treat employees and stockholders fairly." The statement contains no evaluation of the worth of fulfillment of the different aims and does not indicate the interrelationships that may exist among them. If, as is invariably the case, an overall valuation of a utility function for the many features

of corporate aims does not exist, we have to devise methods to give operational meaning to them. In Shubik's example we can select one feature and assume that the firm wishes to maximize it subject to the boundary conditions which require that the other corporate aims meet certain specifications. Thus, we can represent the firm as:

> Maximizing profits subject to maintaining a specific growth program, dividend rate and employment policy which will satisfy stockholders and employees sufficiently that they do not act to change our environment (Shubick, 1961, p. 368).

Alternatively, if the dominant interest of the firm is to take care of its employees, its goals may be stated:

> Maximizing disbursement to employees subject to maintaining a specific growth pattern and dividend policy which will satisfy stockholders (Shubik, 1961, p. 368).

Thus Shubik would have us keep an objective function in the picture and, in addition, add a set of constraints. The objective function would represent the dominant goal of the firm. The constraints would represent those objectives which the firm is intent on satisfying.

An Assumption About the Objective Function

In our picture of the firm we will continue to use profit maximization as the objective function of the firm. We do this for three reasons. First, it is a logical

extension of Maslow's theory. According to Maslow, after all other needs are reasonably well satisfied, the self-actualization need will dominate the actions of the individual. We have shown that there is an analogy between self-actualization in the individual and profit maximization in the firm.

For the second reason, we follow Joan Robinson (1953, p. 590) who said,

> Meanwhile I am inclined to retort to those who grouse about the assumption that the entrepreneur's aim is to maximize profits in the immortal words of Old Bill: "If you know of a better 'ole, go to it."

The figure of speech is apt (Ashley, 1961, p. 96) for the cartoon of Old Bill shows him in a hole which appears to be the target of artillery fire from all directions. Perhaps, if he had scampered away, he might have found a place where he could have dug himself a better hole; his horizon was limited.

Or finally we could follow Professor Scitovsky (1959, p. 59) who said, "We have a vested interest maintaining this assumption--it makes economic analysis so much easier."

A business firm, however, will not be content with using profit just in the objective function. The basic problem is that the distinction between constraints and goals tend to blur. If a firm operates to maximize

profits subject to inviolable constraints, it can be argued that fulfilling the constraining conditions is more important to the firm than is the function to be maximized. A business firm will want to insure that other objectives do not take precedence over a satisfactory profit. This could be accomplished by including a satisfactory profit requirement as part of the system of constraints. It is reasonable to expect, however, that the entrepreneur will be aware of the value of the objective function and can determine if this value satisfies any minimum profit requirements he might have. The imposition of a satisfactory profit constraint would therefore be redundant.

Before leaving this subject of multiple goals, it should be noted that many problems exist in the identification of goals. Some possible goals can be defined precisely and lend themselves to measurement. These include such things as market shares, profits, sales, and balance sheet homeostasis.[3]

Other goals, perhaps no less important to the firm, such as power, survival, socially responsible behavior, etc., can not be defined so precisely and elude efforts toward measurement.

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[3] Balance sheet homeostasis is a concept borrowed from biology. It refers to the firm's attempt to maintain certain balance sheet ratios at a predetermined level.

SECTION IV

THE FIRM AND ITS MARKETS

A second major criticism of our portrayal of the firm concerns the assumptions made about supply and demand. We have essentially assumed pure competition in the demand for final products. Furthermore, we have assumed that the firm, by its own actions, cannot affect the price of its inputs. Here we are using Chamberlin's (1948, p. 6) definition of pure competition, that is "competition unalloyed with monopoly elements." The sole requirements of Chamberlain's definition is that no participant has any degree of control over price. Control over price is essentially eliminated when:

1. There are a large number of participants--enough to insure that any one participant's influence is negligible.

2. Products must be perfectly homogeneous.

A Discussion of the Basic Assumption

The assumption of a constant price in the factor market does not concern us to a great extent, primarily because the assumption conforms closely to reality. Even firms which control a large percentage of a final product market (perhaps even a monopolist) do not tend to be a

large enough user of input factors so that their actions materially affect the price of these factors.

The assumption of pure competition in the output market is of more concern to us. These conditions are approached in some industries such as agriculture. In general, however, this assumption does not conform to reality.

The market structure assumption is not, per se, of prime importance to this study. What is important, is the assumption of a constant price for final products during the mono-periodic production period.

Although pure competition exists in only a few places in our economy, the assumption of a constant price for the period under consideration is not a great departure from reality.

Oxenfeldt (1951, p. 191) notes the following:

> Probably the best reason for using a constant price assumption is that evidence supports Hall and Hitch who studied the price policies of thirty-six firms in England and concluded that "changes in price are frequently very costly, a nuisance to salesmen, and are disliked by merchants and consumers."

Oxenfeldt (1951, p. 189) also points out that large firms such as Sears, Roebuck and Company, and Montgomery Ward are able to publish a retail catalog where prices are constant for periods of up to a year. Joel Dean (1951, p. 457) notes that "its [price rigidity] existence should be recognized."

Although the theoretical reasons for this apparent price stability are not known, a study of oligopoly theory probably sheds the most light on the subject.

Oligopoly and Price Stability

The dominant market in the American economy is oligopoly (Chamberlin, 1950). Oligopolistic markets are characterized by fewness of sellers, restricted entry, and mutual interdependence. Furthermore, prices in oligopoly markets are usually quite stable. Although the reason for this price stability is not known with certainty, it is often attributed to such things as collusion, price leadership, or fear of competitive reactions. An analogy could easily be drawn between the reasons for price stability in oligopolistic markets and human safety needs as postulated by Maslow (1954) and discussed in Section III.

The kinked demand curve

Some economists have used the kinked demand curve to explain this price stability. The oligopolist's demand curve is viewed as having a kink at the point of the prevailing price. The basic assumption is that if you raise price, your competitors will not. If you lower price, however, competitors will tend to follow. Thus the demand curve is thought to be relatively elastic at prices

above the kink, and relatively inelastic at prices below the kink. The distinguishing feature of this demand curve is that it is not differentiable at the kink. There is a "gap" between the value of the left hand derivative and the value of the right hand derivative. This results in a discontinuity of the traditional marginal revenue curve. The curve, therefore, explains stability in spite of variations among the marginal cost curves of different firms in the industry. The kinked demand curve is consistent with profit maximization and the traditional assumption that the firm equates marginal revenue and marginal cost. The kinked demand curve analysis is limited, however, because it does not explain how the prevailing price is reached. The works of Bain, Andrews, and Fellner do, however, shed light on this problem.

Price determination

Bain (1949) insists that oligopolistic firms will set price so as to discourage potential competitors from entering the industry. Thus they are really setting prices so as to maximize long run profits. Bain defines a "limit price" as being the highest common price which the established firms in the industry believe they can change without encouraging at least one new entry into the industry.

Andrews' (1949) theory is based on the "full cost principle." He believes that firms select their prices on

the basis of direct costs plus a standard profit margin to cover overhead expenses. Andrews refers to this price as a "right price." Any variance from this price by the firm will not be profitable. He believes, as does Bain (1949), that a price which is higher than the right price will induce rivals into the industry and eventually cut profits. Prices lower than the right price will not be profitable because the "rightness" of the price results from it representing full costs.

Fellner (1960) uses a bargaining approach to analyze oligopoly prices. He believes that stable prices in oligopoly are the resultant of a type of intraindustry bargaining which is similar to bargaining in a bilateral monopoly. Fellner does not imply that actual negotiations must take place in order to have bargaining. Fellner (1960, p. 54) states that "spontaneous coordination" arises because oligopolistic situations are bargaining situations. "They involve two or more participants who know that what they do affects the policies of others, just as the action of others affects them." In such cases a range is established within which a given price tends to emerge. The absolute limits of this range are set by zero profits for any of the firms. Between these zero profit points lies the bargaining range. The price which is set within

this range will depend on the relative bargaining strengths of firms in the industry.

Bain's "limit price," Andrews' "right price," and Fellner's "bargaining price," help to explain the level of the kink in the kinked demand curve. The important thing for us here, however, is not how the kink got where it is, but rather that price stability does seem to be a fact.

Sales Constraints in the Portrayal of the Firm

The kinked demand curve implies both a unique price and a unique output. If the firm were aware of this unique output, then it would be easy to incorporate this value into our picture. As was the case with our multiple goals, we could simply add a constraint which required that a given amount of the product in question be sold. For the typical firm, the constraint will not take the form of a strict equality constraint. The firm normally reacts to expected demand. It will usually have in mind an amount of any product which it feels it can sell. The firm will normally plan to sell any amount up to this maximum figure. Thus the sales constraints will generally take the form of "less than" inequality constraints.[1]

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[1] In some cases a firm will feel as though it must produce a minimum amount of a certain product in order to maintain a corporate image, customer good will, etc. These types of constraints are generally considered as multiple goal constraints rather than sales constraints.

The preceding discussion was not meant to imply that demand estimation should always be abandoned and replaced by a "right price" and a sales constraint. The introduction of these concepts does, however, serve to focus our picture of the firm.

SECTION V

A RESTATEMENT OF THE PORTRAYAL OF THE FIRM

In our portrayal of the firm we now have an objective (profit) function, a production function, and a set of constraints. These constraints represent alternative goals of the enterprise that must be satisfied as well as sales limitations which must be considered.

The major question to be answered now is: what effect will the addition of these constraints have on the optimal solution derived earlier? Let us make the heroic assumption that these constraints can be quantified, and further that they can be expressed either as functions of the input variables alone, or as functions of the output variables alone. Actually, these assumptions are quite valid for the marketing constraints. These constraints will generally take the form that some function of the output variables is less than a constant. The assumption that all goals of the firm can be quantified has been discussed before and is indeed heroic. To the extent that they can be quantified, however, these goals will generally be a function of either input variables or output variables and not of both. With these assumptions made, we can again solve our constrained maximization problem.

A Revised Entrepreneurial Solution

For the sake of mathematical convenience, we will let A represent the set of indices of the constraints associated with the output variables. B will represent the set of indices of the constraints associated with input variables. Furthermore we shall assume that all constraints (except the production function) are "less than" inequality constraints. Any constraint not in this form can easily be converted by multiplying through by -1.

We can now form our new Lagrange function.

$$L = \sum_{i=1}^{n} p_i x_i - \sum_{j=1}^{m} w_j y_j - S + \lambda_1 [F_1(x_1, . . ., x_n)$$
$$- G_1(y_1, . . ., y_m)]$$
$$+ \sum_{r \varepsilon A} \lambda_r [F_r(x_1, . . ., x_n) + u_r^2 - b_r]$$
$$+ \sum_{s \varepsilon B} \lambda_s [G_s(y_1, . . ., y_m) + u_s^2 - b_s].$$

The necessary conditions for a maximum are:

$$\partial L/\partial x_i = p_i + \lambda_1 (\partial F/\partial x_i) + \sum_{r \varepsilon A} \lambda_r (\partial F_r/\partial x_i = 0,$$
$$i = 1, . . ., n,$$
$$\partial L/\partial y_j = - w_j - \lambda_1 (\partial G/\partial y_j) + \sum_{s \varepsilon B} \lambda_s (\partial G_s/\partial y_j) = 0,$$
$$j = 1, . . ., m,$$

$$\partial L/\partial \lambda_1 = F(x_1, \ldots, x_n) = G(y_1, \ldots, y_m),$$

$$\partial L/\partial \lambda_r \to \lambda_r[F_r(x_1, \ldots, x_n) - b_r] = 0, \quad r\varepsilon A,$$

$$\partial L/\partial \lambda_s \to \lambda_s[G_s(y_1, \ldots, y_m) - b_s] = 0, \quad s\varepsilon B,$$

$$\lambda_r \leq 0, \quad r\varepsilon A,$$

$$\lambda_s \leq 0, \quad s\varepsilon B,$$

$$F_r(x_1, \ldots, x_n) \leq b_r, \quad r\varepsilon A$$

$$G_s(y_1, \ldots, y_m) \leq b_s, \quad s\varepsilon B.$$

Ramifications of the Revised Entrepreneurial Solution

Our primary purpose in stating the necessary conditions for a maximum is to show that we have now lost the neat and simple expressions we had previously prescribed to the entrepreneur. In his search for optimization, he cannot depend on neat slogans such as "utilize all inputs in such a manner that the value of their marginal product equals their market price." Thus, if the economist is going to aid the entrepreneur, he must search for simpler, and more efficient tools.

Before leaving this analysis, we should review several things we have learned from the imposition of additional constraints. Each constraint added to our picture can only decrease (or not change) the absolute amount of profits involved in the optimal solution.

Although this is a mathematical fact, it may come as a surprise to some entrepreneurs who insist that the maintenance of some goal such as market share is essential in maximizing profits.

If the optimal solution could be obtained mathematically, the Lagrange multipliers would represent the opportunity costs of maintaining an objective or a sales constraint at a given level. Thus if a numerical value could be obtained for any Lagrange multiplier, then the entrepreneur could estimate the value of altering the associated constraint requirement by an incremental unit. For example, if the Lagrange multiplier associated with a given sales constraint is zero, then any additional sales effort spent on this output would constitute a wasted resource.

Let us also recognize that there may be no solution to the Lagrange equation. Mathematically we might say that the constraints are inconsistent. Practically we would say that the firm has too many goals and it simply cannot satisfy them all. It is also possible to have a solution to the Lagrange equation which does not meet the minimum profit requirements of the entrepreneur. In these cases it becomes the duty of the entrepreneur to reevaluate the constraints of the firm and to choose one or more to be modified so that a satisfactory solution may be reached.

SECTION VI

THE ENTREPRENEUR AND THE ENTREPRENEURIAL PROCESS

Throughout the first portion of this study we have said quite a bit about the entrepreneur without describing who he was or what he did. This section will attempt to define more clearly the concepts of the entrepreneur and the entrepreneurial process.

The Entrepreneur

The entrepreneur is a manager. He manages the business firm. A business firm may be viewed as an instrument for the transformation of the services of persons and things into completed products. The basic structural unit in an enterprise can be termed as a group.[1] A group is a combination of two or more individuals jointly contributing specialized services which are coordinated to the attainment of a firm's objectives. A complex is a combination of two or more groups jointly contributing specialized services which are coordinated to the attainment of an objective of the firm. A complex differs from the group in two respects. First, the basic units of a complex are groups rather than individuals;

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[1]We are borrowing our definitions from Barnard (1938).

second, the specialized services contributed to a complex are the services of member groups and not directly of individuals. The business firm is the resultant of the combination of individuals into groups, of groups into complexes, of subordinate complexes into superior complexes, and finally into the supreme complex which is the business firm.

Each group and each complex is headed by a manager. These managers are responsible for coordinating the services contributed by the units comprising the groups or complexes which they head. The managers of groups manage the individuals who comprise the groups. The managers of complexes comprised of groups do not directly manage the individuals comprising the groups but only do so indirectly by managing the managers who head those groups. The managers themselves are combined into managerial groups; thus the managers of a combination of groups together with the managers of a complex comprising these groups may be referred to as a managerial group. The same may be said for the managers of subordinate complexes and the manager of the superior complex comprising these subordinate complexes. Individuals who are managers are therefore members of two units--the unit which they head and a managerial unit. This fact relates the managerial superstructure to the structure of groups and complexes and makes possible an

integral whole. This integral, coordinated, pluralistic whole we will call the entrepreneur. It is the entrepreneur's job to optimize. As we stated earlier, economic theorists have not been greatly concerned with how the entrepreneur optimizes.

The Entrepreneurial Process

The twentieth century has seen the rise of another group of analysts, often called management theorists, who are interested in how the entrepreneur optimizes. In fact, that is their whole reason for existence. The approaches taken by management theorists are quite varied. One approach is to divide the entrepreneur's job into managerial functions. Koontz and O'Donnell (1959) state that it is the job of the entrepreneur to plan, organize, staff, and control the business firm. Through these managerial functions, the entrepreneur should coordinate the activities of the firm in such a manner as to best achieve the objectives of the firm.

There is another approach taken by some management theorists. This approach holds that the entrepreneur utilizes his available assets, by operating certain activities in a way which will best achieve some predetermined objective. The various activities of the firm include such things as manufacturing, selling, personnel

administration, accounting, and finance. The assets (liabilities are negative assets) include such things as land, plant, equipment, inventory, accounts receivable, etc. This second approach is more applicable to this study and is the one we shall employ.

Decision-making

Although different management theorists have different basic methods of viewing a firm, most theorists agree that the technique of decision-making pervades the performance of the entrepreneur. Koontz and O'Donnell (1959) note that in order to plan, organize, direct, or control, the entrepreneur must make decisions which affect the operations of the firm.

Haynes and Massie (1961) note that entrepreneurs select among possible activities, and then decide how much emphasis to place on the selected activities. Thus, we may think of the entrepreneur as a decision-maker. He will generally strive to make those decisions which are in some sense optimal.

Etymologically, "to decide" means "to cut off." In its present usage it suggests the coming to a conclusion. It "presupposes previous consideration of a matter causing doubt, wavering debate, or controversy and implies the arriving at a more or less logical conclusion that brings doubt, debate, etc., to an end" (*Webster's*, 1942, p. 1942).

Decision-making involves a conscious choice or selection of one alternative from among a group of two or more alternatives. In making a decision, an individual must become aware of the relevant alternatives, define them, and evaluate them as a basis for choice. Tannenbaum (1950) defines the following steps in the decision-making process.

Awareness of alternatives.--Before making a decision, an entrepreneur should become aware of all alternatives which are relevant to the decision to be made. This, of course, is seldom possible. Often an entrepreneur must depend upon his own limited experience and information. Memory of these is often sketchy and incomplete. It is possible for an entrepreneur to discover relevant alternatives through investigation or by tapping the knowledge of others. This process is, of course, excessively time consuming and does not guarantee complete coverage of all alternatives. For these reasons, it is exceedingly doubtful if most decisions are based upon awareness of all relevant alternatives.

Definition of alternatives.--Once the entrepreneur is aware of alternatives, he must next define each of them. Ideally, this definition involves a determination of all the consequences related to each alternative under consideration. This, of course, can never be fully achieved.

The consequences of various alternatives lie in the future and therefore must be anticipated. Whenever the future is anticipated, uncertainty is present. Uncertainty is present because a decision-maker never has the knowledge to make it possible to accurately determine the nature of the consequences which will follow upon his choice of a given alternative, assuming all other related elements remain constant. In addition, all other related elements will probably not remain constant.

Evaluation of alternatives.--After an entrepreneur has become aware of certain alternatives and consequences associated with these alternatives, he must make a choice among them, that is, he must make a decision.

There are two basic types of decisions any individual must make. Some of these decisions (a small proportion) relate to the individual's system of values. They determine his ultimate ends. All other decisions are directly or indirectly related to means for the attainment of these ultimate ends. In choosing among alternatives, an entrepreneur will attempt to make a selection, within the limits of his knowledge, which will maximize results (the degree of attainment of the relevant end) at a given cost or which will attain given results at the lowest cost. Thus, the individual has a criterion to guide his choice. This criterion is similar to the

economist's concept of an opportunity cost. Joel Dean (1951) defines opportunity costs as profits foregone because of the exclusion of a particular alternative. An entrepreneur can very seldom place a dollar value on the opportunity cost of any alternative. Nevertheless, the decision-making process must take place in this framework. Obviously, the opportunity cost of an alternative which is presently being used is zero. There can be no foregone profit associated with an alternative which is being used. In order for any alternative to be valuable, it must have a positive opportunity cost. In other words, the firm must be foregoing something of value by not utilizing the foregone alternative. The rational entrepreneur, in the decision-making process, will choose that alternative which has the highest opportunity cost. The opportunity cost of each alternative is relative to the chosen alternative. If the entrepreneur chooses the most profitable alternative, then the opportunity costs of all excluded alternatives must become negative. The implication being that since the best alternative is not among the excluded set, a selection of an element of this set would reduce the entrepreneur's profits.

The entrepreneurial sphere of discretion

With respect to any given problem which requires a decision, the entrepreneur (any-individual) may have many

feasible alternatives among which to choose. Following Tannenbaum (1950), we shall define an entrepreneur's sphere of discretion as the set of all feasible alternatives. The factors which restrict, restrain, or limit the exercise of discretion to available alternatives are referred to as "constraints." Decision-making, then, is judgment exercised within constraints.

Type I and type II decisions

Tannenbaum describes two basic types of decisions any individual must make. Some decisions (usually a small minority) relate to the individual's system of values. They determine his ultimate ends. All other decisions are directly related or indirectly related to means for the attainment of these ultimate ends. It is important for us to make a distinction between these two types of decisions. We shall refer to the type of decisions which relate to an individual's system of values as type I decisions. Those decisions which relate to the means for the attainment of the ultimate ends will be defined as type II decisions.

It is important to realize that type I decisions[2] must be made first. Once they are made, they effectively

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[2]The distinction between type I and type II decisions tends to be relative. In Section I, we noted that any optimizing procedure that incorporates the assumption of a mono-periodic production period is actually a suboptimizing procedure. Business firms

serve as constraints upon all type II decisions. If, for example, the entrepreneur decides (makes a type I decision) he wants to sell a certain minimum quantity of a particular product, then this limits or constrains the type II decisions he can make in an effort to maximize his objective function.

It is also possible that the decision-maker can make such a variety of type I decisions that there are no type II alternatives which will satisfy all type I decisions. In this case, the entrepreneur must reevaluate his type I decisions, and alter one or more of them so that a feasible solution can be found.

The Iterative Nature of the Entrepreneurial Process

Litchfield (1956) notes that the entrepreneurial (administrative) process is a cycle of action which includes decision-making, programming, and reappraising. Certainly an entrepreneur is not an omniscient individual. He cannot immediately see the consequences of all his actions, thus the cyclical process. Typically, the problem

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have long-run goals and objectives. The requirements and objectives of the firm for a given mono-periodic production period can not be set in isolation. Consideration must be given to the effect of these decisions upon the long-run position of the firm. Thus, short-run requirements are usually made with the aim of achieving long-run objectives. The type I decisions of a short-run model may be the type II decisions for a long-run model.

faced by an entrepreneur is not that of finding a completely new solution, but rather improving on an existing solution. Thus an entrepreneur may start with a feasible solution. He then evaluates the opportunity costs of the excluded alternatives and checks to see if a better solution is possible. If a better solution is possible, he then selects the best alternative and incorporates it into his solution, checks the new solution for feasibility, and then programs the new solution (puts it into operation). He then repeats the cycle. If the entrepreneur ever reaches a point where all of the excluded alternatives have non-positive opportunity costs, he can assume that he can do no better.

Let us define more precisely what we mean by a managerial or entrepreneurial solution. An entrepreneur chooses among feasible alternatives. Typically he does not choose one alternative to the exclusion of all others. He more likely will choose a small set of alternatives from among a large set. Furthermore, some of the chosen alternatives will receive more emphasis than others. It is convenient to think of alternatives in the excluded set as receiving no emphasis.

Thus the set of feasible alternatives, together with the amount of emphasis each is receiving at a particular time, determine the entrepreneur's solution or program.

We may combine the thoughts of Tannenbaum and Litchfield to define an entrepreneurial process. This process is illustrated schematically in Figure 1. Let us note that the previously defined concept of an "activity" can easily fit into the framework of Tannenbaum's "alternative."

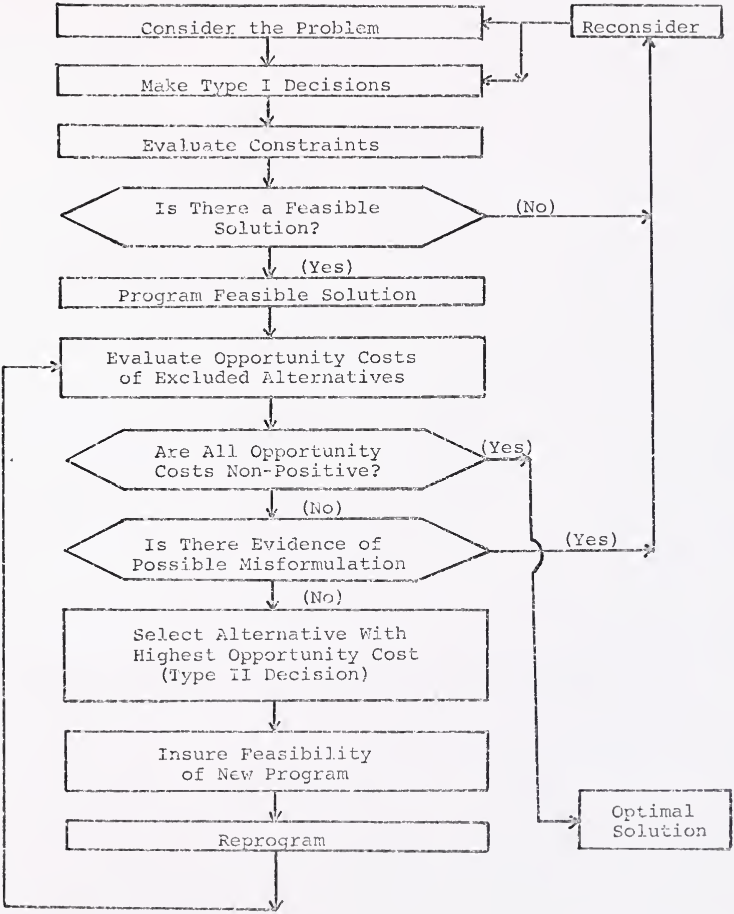

FIGURE 1--SCHEMATIC REPRESENTATION OF THE ENTREPRENEURIAL PROCESS

SECTION VII

A REEVALUATION OF THE PRODUCTION FUNCTION

In our analysis up to this point, we assumed the production function to be given. We did not concern ourselves with how this function was derived, but only used this function to characterize the optimum production alternatives when considered in relation to the markets in which the firm must buy and sell, and the multiple goals of the firm. Historically, this has been the position taken by economic theorists. This conventional theory is often justified on the grounds that the analysis of the firm is but one step in the analysis of economic markets.

Suppose, however, that we are interested in the firm per se. Suppose we are interested in aiding management in determining how to solve its optimization problem. We cannot now assume that the derivation of the production function is irrelevant. Instead we must examine more critically the traditional assumptions about the production function.

The choice variables in the traditional production function are generally conceived as time rates of consumption of various inputs and time rates of production of various outputs. The choices of production and

consumption are, of course, dependent, otherwise output would be chosen very large and input very small.

Business firms are operated by entrepreneurs. Entrepreneurs are human. We must accept the fact that they do not possess perfect knowledge and therefore are not fully aware of the firm's production function. What then, do entrepreneurs do? We have stated that entrepreneurs are primarily decision-makers. Following Tannenbaum, we have defined decision-making as a choice among feasible alternatives. Thus we can see that the concept of the traditional production function is really not relevant to the modern firm. Normally, the choice is among various feasible alternatives. These feasible alternatives may be thought of as "different ways of doing things." Each alternative implies its own characteristic pattern of input and output rates.

The Concept of a Process

The decisions made by the firm do not deal directly with levels of input and output but are more concerned with the choices among technically feasible processes. We shall define a process as a "way of doing something." A process is a set of ratios among rates of consumption of various inputs and rates of production of various outputs. Thus a firm makes a choice among a number of processes.

The firm is not restricted to the use of only one process. It can normally utilize several processes simultaneously. Furthermore we shall assume that most processes can be operated at a range of levels. By level of a process we mean the time rate of consumption of all inputs and the time rate of production of all its outputs in the same proportion.[1]

When we were discussing the entrepreneurial process we noted that the entrepreneur normally chooses a small set of alternatives from among a large set. Furthermore we

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[1]Implicitly we have assumed linearity. Initial thought was to include an appendix to justify this assumption. This was not done for three reasons. First, there are many articles and books in the economic literature which illustrate examples of the application of linear models to economic problems. For example, see Bowman and Fetter (1959). Second, the entrepreneurial model presented in later sections of this study is formulated in terms of "balance sheet accounts" and "business transactions." For the most part, it is obviously linear. Third, in the implementing part of this study effort was made to determine if this assumption had any detrimental effects on the implementing model. This was not found to be the case. The concept of linear programming will be discussed in subsequent sections. It should be pointed out here, however, that the phenomenon of decreasing marginal returns is encountered in a linear programming analysis of the firm. Baumol (1959, pp. 270-274) and Wu and Kwang (1960) give a comprehensive comparison of diminishing productivity in the traditional economic analysis and diminishing productivity in the linear programming analysis. Baumol (1959, p. 281) states, ". . . the ordinary law of diminishing returns is compatible with linear programming, i.e., the marginal yield to increased use may decline, provided the employment of other factors remains unchanged, . . . but the decreases characteristically occur in discontinuous jumps."

stated that some of the chosen alternatives will receive more emphasis than others, and that alternatives in the excluded set could be thought of as receiving no emphasis. Thus the set of feasible alternatives, together with the amount of emphasis each is receiving, define the entrepreneur's solution or program.

We can now be more specific and say that the entrepreneur selects a small set of processes from among a large set. The level of a process corresponds closely to the degree of emphasis. All processes in the excluded set can be thought of as being utilized at a zero level. Thus the set of processes, together with their level of utilization, define the entrepreneur's solution or program.

A process can represent any activity within the firm. It may represent sales effort, personnel relations, research and development or anything else a firm might do. The definition of a process implies something more specific than an activity undertaken by the firm. It is the way or the method that a firm undertakes an activity.

One of the most common activities of a firm is that of manufacturing a product. It is essential, however, to distinguish between a product and a process. A product is an economic good or service which is sold on the market for a particular price. A process is a way of manufacturing a particular product. Normally there will be a number of

technically feasible processes which will produce the same product. The same is true for most other activities undertaken by the firm. Thus, the number of processes will always be greater than or equal to the number of activities. It is possible in some cases to have a large number of processes for a given product. Usually, the number of processes available is small. We shall assume that the entrepreneur is free to utilize several process simultaneously so long as he does not violate his sphere of discretion.

We have stated that there is a constant price associated with each input and each output. Since a process is a set of ratios among rates of consumption of various inputs and rates of production of various outputs, it must be true that associated with each process is a value which we shall call the net contribution to profit and overhead. Net contribution represents the difference between the additional revenue received by and the additional cost incurred by the utilization of one additional unit of any given process. Under our assumption of linearity, net contribution is a constant and is independent of the level of utilization of any process. It would be incorrect to refer to this new parameter as profit. The firm does incur a certain amount of fixed

costs (which may be called overhead) and technically profits are not obtained until the net contribution is greater than overhead.

Since a firm will normally employ several processes, it follows that the firm's total consumption of resources and total production of outputs will be the sum of the quantities of factors consumed by the various processes and the sum of the products created by the various processes. A change in the quantity or proportion of inputs consumed or outputs produced can only result from a change in the levels at which the various processes are operated. The firm cannot alter directly the quantities of inputs or outputs, but can only change these factors indirectly by means of changes in the level of various processes.[2] It should be the goal of the entrepreneur to choose those levels of the various processes which will maximize his own objectives while at the same time not violate his sphere of discretion.

Processes and the Production Function

The introduction of the concept of a process has forced us to reexamine the traditional production function. The typical entrepreneur is not aware of an "optimal"

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[2]We shall assume that all processes must be operated at non-negative levels.

production function on which he must operate. He is much more aware of the limitations or constraints on his selection of processes. The entrepreneur has available certain fixed and variable factors of production. The entrepreneur uses these inputs (by operating processes) to create outputs. In doing so, the entrepreneur selects among various alternative processes. He is not free to operate any process at any level. He is constrained by the availability of certain factors of production. We shall call these constraints the direct production constraints.

We shall refer to all other constraints placed upon the entrepreneur as indirect production constraints. The direct and indirect production constraints together define the entrepreneur's sphere of discretion.

We have now redefined our production function. Instead of the continuous production function of the traditional economists, we have a production function that consists of two parts; a set of processes, and a sphere of discretion.

Processes and the Objective Function

The introduction of the concept of a process has also forced us to reformulate our objective function in terms of processes. This is easily done mathematically as follows:

$$P = \sum_{i=1}^{n} c_i x_i - S.$$

x_i represents the level at which process i is utilized in any solution. c_i represents the net contribution of a unit of process i. S represents the fixed or sunk costs which must be paid by the firm regardless of the rate of output. It is the duty of the entrepreneur to select those processes and operate them at the correct level to maximize this objective function.

SECTION VIII

THE RELATIONSHIP BETWEEN THE PORTRAYAL OF THE FIRM AND MATHEMATICAL PROGRAMMING

Mathematically, any problem which seeks to maximize (or minimize) a numerical function of one or more variables (or functions) when the variables can be independent or related in some way through the specification of certain constraints may be referred to as an optimization problem. The methods of differential calculus have long been applied to optimization problems in the theory of the firm. In fact, traditional economic theory of the firm is framed in the methods of differential calculus.

In the last twenty years there has been a large growth of interest in a new class of optimizing problems, referred to as programming problems, which are usually not amenable to solution by classical methods of calculus.

The General Programming Problem

The general programming problem can be formulated in the following way. The objective is to determine values for n variables $x_1, x_2, \ldots, x_n$ which satisfy the m inequalities or equations

$$g_j(x_1, x_2, \ldots, x_n)\{\leq, =, \geq\}b_j, \quad j = 1, \ldots, m,$$

and in addition, maximize (or minimize) the objective function

$$z = f(x_1, x_2, \ldots, x_n).$$

The constraints are assumed to be specific functions, and the b_j are assumed to be known constants. One and only one of the signs $\{\leq, =, \geq\}$ holds for each constraint, but the sign may vary from one constraint to another. The values of m and n need not be related in any way. m can be equal to, less than, or greater than n. Usually, some or all of the variables are restricted to be non-negative.

Linear Programming

The real impetus for the growth of interest in programming problems came in 1947 (Hadley, 1964, p. 14) when George Dantzig devised the simplex algorithm for solving the general linear programming problem. If,

$$g_j(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} a_{ji}x_i, \quad j = 1, \ldots, m,$$

$$f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} c_i x_i,$$

where a_{ji} and c_i are known constants, the programming problem is said to be linear provided that there are no other restrictions except perhaps the requirement that some or all of the variables must be non-negative.

Usually in the formulation of the general linear programming problem, it is specified that each variable must be non-negative, i.e.,

$$x_i \geq 0, \; i = 1, \ldots, n.$$

This form is most convenient when making numerical calculations. Thus a linear programming problem seeks to determine non-negative values of the n variables x_1, x_2, . . ., x_n, which satisfy the m constraints

$$\sum_{i=1}^{n} a_{ji} x_i \{\leq, =, \geq\} b_j, \quad j = 1, \ldots, m,$$

and which optimize the linear function

$$z = \sum_{i=1}^{n} c_i x_i .$$

It is often convenient to express the general linear programming problem in vector form, i.e.,

$$\max z = cx,$$

$$\text{s.t.} \; \sum_{i=1}^{n} a_i x_i \{\leq, =, \geq\} b.$$

c and x are n component vectors and may be represented as

$$c = (c_1, c_2, \ldots, c_n),$$

$$x = [x_1, x_2, \ldots, x_n].$$

c is generally referred to as the price vector and x as the program vector.

a_i and b are m component vectors and may be represented as

$$a_i = [a_{1i}, a_{2i}, \ldots, a_{mi}],$$

$$b = [b_1, b_2, \ldots, b_m].$$

The a_i are called activity vectors and b is generally referred to as the requirements vector.

Linear Programming and the Portrayal of the Firm

The typical linear programming formulation is almost identical to our modified picture of the firm. The objective function is the linear programming formulation does not contain the term -S. In other words, there is no provision for fixed or sunk costs in the linear programming model. It has been emphasized, however, that fixed costs are relevant only to the decision of whether to operate the firm or shut it down.

The linear programming activity vector is identical to our concept of a process. It is a set of ratios among rates of consumption of various inputs and rate of production of various outputs. The linear programming decision variables are identical to our concept of level of a process. The program vector (vector of decision variables) is identical to our concept of an entrepreneurial solution. The prices in the linear programming formulation

correspond to our net contribution values. The linear programming constraints define our entrepreneur's sphere of discretion.

The Simplex Process

The formal method of solution of linear programming problems is not, per se, of great interest here. It is interesting to note, however, the similarity between the iterative mathematical solution process of linear programming and the entrepreneurial process as outlined by Tannenbaum and Litchfield, and discussed in Section VI.

The simplex method for the solution to linear programming problems is an iterative procedure which reaches an optimal solution in a finite number of steps or provides an indication that there is an unbounded solution.[1] If the problem has an optimal solution, the optimum value of z must be finite. Let us assume that

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[1]An unbounded solution indicates a problem in which the value of the objective function can be made infinitely large. Such solutions are not expected in the real world and generally indicate a misformulation of the problem. As Gue and Thomas (to be published) have said, ". . . if the reader is aware of a profit maximization problem where the objective function grows without bound, he is requested to write the authors immediately with the details of the problem."

the linear programming problem has been converted to the standard form,[2]

$$\max z = cx,$$

$$\text{s.t.} \sum_{i=1}^{n} a_i x_i = b,$$

$$x \geq 0.$$

The two following theorems can easily be proved (Hadley, 1964, p. 31).

1. If the problem has an optimal solution, at least one basic[3] feasible solution will be optimal.

2. If we have a basic feasible solution which is not optimal, it is possible to reach an optimal basic solution in a finite number of steps by changing just one of the basic variables at each step, or to obtain ultimately an indication of an unbounded solution.

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[2]For mathematical convenience, it is advisable to convert inequalities into equalities. This is done by the addition of slack and surplus variables. *Slack variables* are introduced into "less than" inequalities and represent the difference between the maximum available resource, b_j, and the amount actually used. *Surplus variables* are introduced into "greater than" inequalities and represent the amount by which a minimum requirement, b_j, is exceeded. It is also usually convenient to have all components of the requirements vector positive. If necessary, this can easily be accomplished by multiplying a constraint equation by -1.

[3]A basic solution is a solution that has no more than m variables different from zero where m represents the number of constraints.

The simplex method begins with an initial basic feasible solution. It then computes the opportunity costs[4] for all vectors (process) not in the basis. If all excluded alternatives (vectors not in the basis) have non-positive opportunity costs, the simplex method concludes that the present solution is optimal. If one or more of the excluded alternatives have a positive opportunity cost, the simplex method checks for the possibility of a misformulated problem (unbounded solution). If there is no indication of an unbounded solution, the simplex method chooses the excluded alternative with the largest opportunity cost to enter the basis. The simplex method then selects a process to be removed in such a way as to insure feasibility of the new solution. The procedure is then repeated until an optimal solution is reached.

There are many cases where the simplex method must begin with a solution which contains artificial processes. The simplex method then follows an iterative procedure to remove the artificial vectors and thereby obtain a real basic feasible solution. In some cases this will not be possible. The simplex procedure will indicate when a problem is formulated so that no feasible solution may be found. A flow chart of the simplex process is shown in Figure 2.

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[4]Actually the simplex method computes an approximation to the opportunity cost.

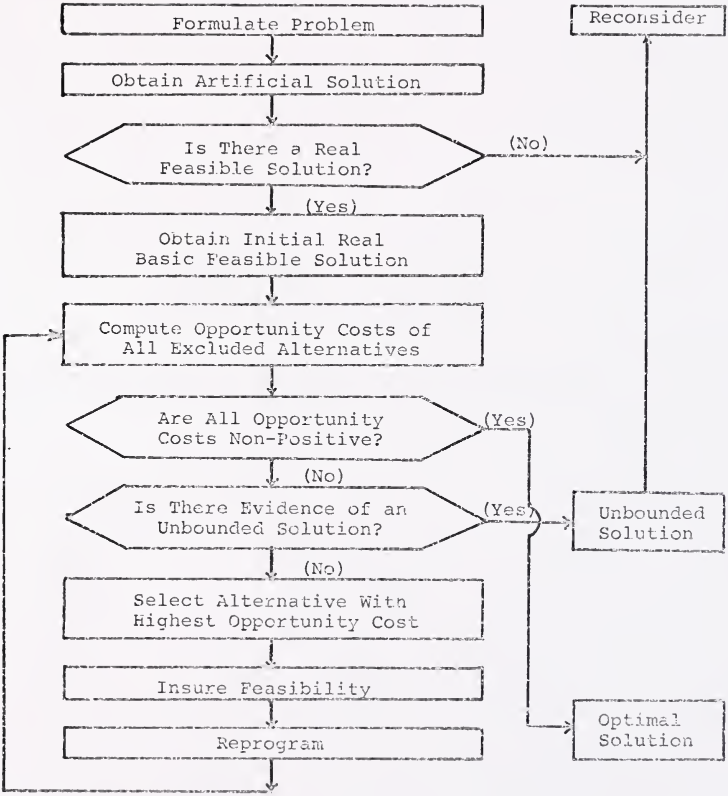

FIGURE 2--SCHEMATIC REPRESENTATION OF THE SIMPLEX METHOD FOR SOLUTION OF LINEAR PROGRAMMING PROBLEMS

Notice the similarity between the flow chart of the simplex method and the flow chart of the entrepreneurial process as shown in Figure 1.

The type II decisions of the entrepreneur correspond closely to the selection of basis vectors in the simplex method. The entrepreneur's type I decisions are reflected in two places in the linear programming process. First in the selection of the objective function, and second in the selection of some components of the requirements vector.

SECTION IX

THE PORTRAYAL OF THE FIRM: AN ACCOUNTING VERSION

The typical linear programming problem is normally formulated in such a way as to obtain computational efficiency in solving the optimizing problem under consideration. If we are to focus more sharply our picture of the firm, however, we should (for the purpose of this study) concern ourselves less with mathematical (computational) efficiency and more with the concepts used by an entrepreneur in operating his business. Previously we have stated that one view of management theory suggests that the entrepreneur utilizes his available assets, by operating processes, in a way which will best achieve some predetermined objective. We stated that the various processes of the firm include such things as manufacturing, selling, personnel administration, accounting, and finance. We also noted that the assets (liabilities are negative assets) include such things as land, plant, equipment, inventory, accounts receivable, etc. Now we would like to be more specific about our concepts of assets and liabilities, and introduce other terms such as balance sheet, net income, business transaction, and double-entry bookkeeping. In short, we want to borrow some terms from the accountant.

Accounting furnishes a good part of the data used by management in making decisions and directing operations. One branch of accounting, administrative or managerial accounting, is based upon the concept of accounting as a method of management or as a tool by which managerial effectiveness is enhanced. Thus, entrepreneurs tend to think in an accounting framework, and the accountants are more and more tending to utilize a framework which enhances managerial effectiveness. We will not be content to use the accountant's framework entirely, however, but we must modify it to some extent so that it will correspond more closely to our picture of the firm.

The Accountant's Balance Sheet

First let us look at a simplified version of the accountant's balance sheet. The balance sheet contains an inventory of the several kinds of assets to which the firm has title and the liabilities which the firm has incurred. Traditional accounting statement generally report assets and liabilities at their cash value. The cash balance is perhaps the most precise valuation on the balance sheet. It is important mainly with respect to what can be done with it rather than the absolute amount held at balance sheet time. Other assets, like receivables and securities, are reported at an exact amount expected to be realized as

the business continues normal operations; this amount may be cost or a value less than cost. Inventories are held for ultimate sale, so the dollar valuation of inventory is the number of cost dollars expected to be recovered from future sales. Dollars tied up in plant and facilities must likewise be recovered as revenue dollars; the balance sheet reflects the dollars of original outlay not yet so recovered. Offset against these costs not yet realized is a list of claims that must be satisfied in the future. These, of course, are the liabilities of the firm.

Balance sheet accounts

In order to make better comparisons and judgments, the accountant classifies the information on the balance sheet. He groups like things together, starting with a basic division of financial elements into assets and liabilities, and then proceeds with common subgroups. A few subclassifications commonly found in a balance sheet are discussed below.

Current assets are those resources now available, claims for payments of cash in the near future, investment in marketable securities, and goods which are expected to be converted to cash as a result of normal operations in the near future. Conversion of assets is a primary objective of management. The accountant, therefore, attempts to report them at realizable value. Accounts

receivable are reviewed with the idea of reporting only those which appear to be collectable. If expected realization on inventories has fallen below historical cost, the accounting valuation is reduced to reflect as accurately as possible the probable recovery. The current asset classification is subdivided in many ways depending on the situation. Three subclassifications found on most balance sheets, however, are *cash*, *accounts receivable*, and *inventory*. The general inventory classification may be divided into finished goods inventory and raw materials inventory.

Fixed assets include both tangible property and intangible rights which will be used for a number of periods, or from which some benefit will be derived for a comparatively long time. Fixed tangible assets are sometimes grouped together into one item, *plant and equipment*. More often, however, descriptive titles are used for each type of tangible assets. *Land*, *buildings*, and *equipment* are three common subclassifications found on most balance sheets. Use is the key to classification of fixed assets.

The accounting procedure for fixed assets provides for some portion of cost to be deducted from the income of each period and charged to the particular asset. The remainder of cost appears on the balance sheet as the

accountant's valuation of the asset, a valuation which has no necessary relation to the current market value of the asset. Because this assignment of cost to periods is a matter of judgment, the original cost of fixed assets as well as the accumulated cost charged off are usually separately reported on the balance sheet (or in a supporting statement).

The assumption is usually made that the firm will endure longer than any particular asset. The entire cost, therefore, of all fixed assets (but no more than cost) will be charged against the revenue of some period (or against revenues from the sales of the asset). In a sense fixed assets may be thought of as being reported at realizable value, just as current assets are, but the term of realization is much longer and may involve a number of future periods instead of just one.

Current liabilities are those debts which mature in the near future and whose liquidation requires the expenditure of current assets. In the case of both assets and liabilities, "current" normally refers to the next operating cycle. *Accounts payable* to trade creditors, *notes payable* to financial institutions, *accrued wages*, *accrued taxes*, etc., make up the current liabilities.

Fixed liabilities, sometimes called long-term debt, are those liabilities which mature sometime in the future

(beyond the current period). That part of long-term debt which falls due within the current period is normally reclassified as a current liability unless some provision has been made to repay (or refund) the debt without a drain on current assets.

Owner's equity is normally reported without regard to individual owners but classified by nature and source of capital. Capital invested by the owners is reported under two headings: par or stated value of the shares issued is reported as *capital stock*, the excess of the investment over par or stated value of the shares is reported as *paid-in* or *capital surplus*. The increase in equity resulting from profitable operations and not distributed to stockholders is reported as *retained earnings*.

A typical balance sheet is shown in Table 1.

Business Transactions

Next let us consider the concept of a business transaction. When the business firm enters into a transaction with other entities, the kind of property owned, or the amount and nature of debts or money obligations, or the owners' equity is changed. A transaction is an exchange of things having monetary value. Many transactions involve an outside party, but transactions can

TABLE 1--HYPOTHETICAL BALANCE SHEET

Account Number	Account Name	Valuation
	Current assets:	
101	Cash	20,000
102	Accounts receivable	100,000
103	Inventory (finished goods)	100,000
104	Inventory (raw materials)	60,000
		280,000
	Fixed assets:	
121	Land	50,000
122	Building	100,000
122a	Accumulated depreciation	(40,000)
123	Equipment	122,000
123a	Accumulated depreciation	(60,000)
		172,000
	Total assets:	452,000
	Current liabilities:	
201	Accounts payable	50,000
202	Accrued wages	10,000
203	Accrued taxes	4,000
		64,000
	Long-term debt:	
221	Bonds payable	100,000
		100,000
	Stockholders' equity:	
301	Common stock	240,000
302	Retained earnings	48,000
		288,000
	Total liabilities:	452,000

Source: See Section IX, *The Accountant's Balance Sheet*, for source and explanation of data.

occur solely within the firm. Any transaction affects two or more accounting elements within the firm. For our purposes, the minimum necessary requirements for a transaction is that it requires a "double-entry" by the firm's accountant. A proprietor's investment of money in the business, for example, is a transaction between the firm and the owner. The effect on the firm is to increase its assets and its capital by the same amount. The consumption of fuel oil in the office furnace is likewise a transaction. Its effect is to diminish the assets of the firm and substitute an expense of equal amount. The sale of a product is still another transaction. Its effect might be to increase accounts receivable and reduce finished goods inventory by the same amount. A transaction of "collection" might then decrease accounts receivable and increase cash.

We have noted that any transaction affects two or more accounting elements (or simply accounts) within the firm. The balance sheet entries previously defined are accounting elements. The most important objective of accounting, from a financial viewpoint, is the measurement of business income or profit. This purpose requires that revenues (incomes) and costs (expenses) be accounted for. Revenue accounts are broken down to show nature of revenue (sales, interest, etc.), source of revenue by product, or

to any other extent which may be useful to management. Cost accounts are usually separated by nature of expenditure.

Process illustrations

Table 2 illustrates the concept of a transaction[1] (although not in the nice debit and credit form normally used by accountants). Six typical transactions are illustrated as follows:

1. *Manufacturing* of one unit of a product. The firm converts raw materials, equipment (per-unit depreciation is assumed), and labor into finished goods inventory. Inventory is valued at cost, therefore final goods inventory is increased by $70. Raw material inventory is decreased by $40, and the value of fixed equipment is decreased by $5. The firm has also incurred labor expenses of $25 (which have not yet been paid).

2. *Selling* one unit of product. The firm has sold a unit of finished goods inventory for $100 on credit. Accounts receivable have increased by $100, final goods inventory has been decreased by $70. The firm has received $100 of revenue from sales and the cost of goods sold was $70.

3. *Collection.* The firm has converted accounts receivable into cash.

4. *Payments.* The firm has paid the wages for part of the labor it utilized in production.

5. *Purchases.* The firm has purchased (for cash) additional raw materials so that additional product can be made.

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[1] For convenience, the chart of accounts in Table 2 has been abbreviated.

TABLE 2--HYPOTHETICAL PROCESS ILLUSTRATIONS

Account Number	Account Name	Process Number 1	2	3	4	5	6
101	Cash			100	-25	-40	-26
102	Accounts receivable		100	-100			
103	Inventory (final goods)	70	- 70				
104	Inventory (raw materials)	-40				40	
121	Land						
122	Other fixed assets	- 5					
201	Accounts payable						
202	Accrued wages	-25			25		
221	Long-term debt						
301	Owners' equity						
401	Sales revenue		-100				
402	Other revenue						
501	Cost of goods sold		70				
502	Other expenses						26

Source: See Section IX, *Business Transactions*, for source and explanation of data.

6. *General* and *administrative* expense. The firm has paid for general and administrative expenses which it used throughout the period under consideration.

The concept of a transaction is very similar to the concept of a process previously defined, and may be thought of as such. It is a set of ratios among rates of consumption of various inputs and rates of production of various outputs. Indeed, we would like to think of a process in terms of a financial transaction. We have noted that processes are the alternatives from which entrepreneurs must choose in the decision-making process, and as Brown (1958, p. 101) has said, "All (business) decisions are financial decisions."

Brown justifies his statement this way:

> All decisions are financial, either because they directly affect the expenditure of money, or because they indirectly affect expenditures by consuming or disposing of effort, facilities, or material, all of which cost money. A decision to improve toilet and locker facilities may begin from a proper concern for the health and comfort of employees, but it requires an expenditure, and every expenditure affects the earnings of the enterprise. All decisions are financial because you cannot conceive of one that does not affect, in one way or another, and in some degree, the earnings of the enterprise.
> . . . to these decisions, the engineer, the production man, the merchandiser, and perhaps others, must all contribute. What they contribute are facts--the facts that each is skilled to obtain. To the facts so contributed, moreover, someone must apply reason--intelligence--so as to give them their proper weight, evaluate them, and relate them properly to each other. This is the material--the foundation--of decision. This is the financial approach.

> . . . each of these men has a primary operating function, but each must approach that function with a financial viewpoint because his decision will affect the earnings of the enterprise. In that sense, each is a financial man.

Let us assume that the ratios shown in Table 2 represent the respective processes when they are being operated at the unit level. For example, if process 4 is operated at a level of 10, the finished goods inventory will be increased by $700, raw materials inventory will be decreased by $400, etc.

Mathematical notation

It is convenient at this point to reintroduce some mathematical notation. Suppose we assign a number j to each of the firm's accounts and assume that j takes on integer values from 1 to w*. We will then group the accounts so that as j varies as shown below, the following type of accounts will be assigned the number represented by j.

Current assets	$1 \leq j \leq p^*$
Fixed assets	$p^* + 1 \leq j \leq q^*$
Current liabilities	$q^* + 1 \leq j \leq r^*$
Long-term debt	$r^* + 1 \leq j \leq s^*$
Owners' equity	$s^* + 1 \leq j \leq t^*$
Revenues	$t^* + 1 \leq j \leq u^*$
Expenses	$u^* + 1 \leq j \leq w^*$

$$1 \leq p^* \leq q^* \leq r^* \leq s^* \leq t^* \leq u^* \leq w^*$$

Let us also assume that there are n processes and we will let i be the index of process numbers so that $1 \leq i \leq n$.

Let x_i represent the level of utilization of process i, and x, the program vector be

$$x = [x_1, x_2, \ldots, x_n].$$

a_{ji} will represent the net effect on the "j"th account by utilizing process i at a unit level. Any process can be represented by the process vector

$$a_i^1 = [a_{1i}^1, a_{2i}^1, \ldots, a_{w*i}^1],$$

The process matrix A^1, will then be

$$A^1 = (a_1^1, a_2^1, \ldots, a_n^1).$$

We will utilize this notation after we have considered another concept utilized by the entrepreneur.

Business Income

Let us briefly investigate a simplified example of a typical income statement of a firm. The income statement is a summary of the changes in equity which result either from transactions between the owner and the firm or from transactions entered into by the firm for profit. Net income is the difference between realized revenues and expired costs for the period under consideration. The

simplest arrangement of an income statement is a two-part summary with all revenues reported first and then all expired costs deducted.

Mathematically, net income can be represented in two forms;

Form I, $(A^1x)^T d^1$ = net income,

$d^1 = [d^1_1, d^1_2, \ldots, d^1_{w*}]$,

$d^1_j = 0, \qquad j = 1, \ldots, t*,$

$d^1_j = 1, \qquad j = t* + 1, \ldots, u*,$

$d^1_j = -1, \qquad j = u* + 1, \ldots, w*.$

Form II, $(A^1x)^T d^2$ = net income,

$d^2_j = 1, \qquad j = 1, \ldots, q*,$

$d^2_j = -1, \qquad j = q* + 1, \ldots, t*,$

$d^2_j = 0, \qquad j = t* + 1, \ldots, w*.$

Form I is more representative of the typical income statement, however form II may, in some cases, be more useful for entrepreneurial discretion.

Let us consider form II a bit closer. Since the last $(w* - t*)$ components of d^2 are zero, we can express form II in a more convenient way. Suppose we introduce new vectors a^2_i and d^3. a^2_i is identical to the vector a^1_i except that it contains only the first $t*$ components

of a_i^1. d^3 is likewise identical to d^2 except that it also contains only the first t* components.

$$a_i^2 = [a_{1i}^2, a_{2i}^2, \ldots, a_{t*i}^2],$$

$$a_i^2 = [a_{1i}^1, a_{2i}^1, \ldots, a_{t*i}^1],$$

$$d^3 = [d_1^3, d_2^3, \ldots, d_{t*}^3],$$

$$d^3 = [d_1^2, d_2^2, \ldots, d_{t*}^2].$$

Net income may now be expressed as

$$(A^2x)^T d^3.$$

Rearranging, we find

$$x^T A^{2T} d^3 = \text{net income}.$$

Let z represent net income and let c^T be an n component vector so that

$$A^{2T} d^3 = c^T.$$

Thus,

$$z = cx = \text{net income}.$$

We have now expressed net income in the form of the linear programming objective function. Under our assumption of profit maximization, this is the entrepreneur's objective function.

Suppose we let y^1 be a t* component vector representing the exact amount in each of the t* balance sheets

accounts at the beginning of the period. y^2 would be a t* component column vector representing the balance sheet accounts at the end of the period. Let us define y to equal A^2x. y represents the net changes (during the period) in all balance sheet accounts except the owners' equity account. It does not reflect the entry of closing net income into owners' equity. Although this may seem abstruse here, it will serve to illustrate concepts used later in this study.

SECTION X

THE PORTRAYAL OF THE FIRM: A MODIFIED ACCOUNTING VERSION

The preceding section of this study has tended to be more of a financial or "bookkeeping" picture of the firm. It did not particularly help us understand the concept of entrepreneurial discretion under constraint. In order to gain a better picture of this process, we must modify our concepts of balance sheets, income statements, and business transactions.

The Entrepreneurial Balance Sheet

First let us consider the balance sheet. In order to allow for entrepreneurial discretion, the major asset classifications on our balance sheet should now be *variable* and *non-variable assets*, rather than current and fixed assets. Likewise, the liabilities should be divided into *variable* and *non-variable liabilities*.

The key to distinguishing between variable assets and non-variable assets is the type of entrepreneurial discretion involved in utilizing the asset. For variable assets, the entrepreneur has a range of discretion in

deciding how (in what way) an asset should be used and how much[1] (in terms of quantity or value) of the asset to be utilized.

For non-variable assets, the entrepreneur still must decide how the asset is to be utilized, but he has a given quantity available for utilization whether he uses it or not.

Variable assets in our new balance sheet correspond closely to the current assets of our old balance sheet; they give us little concern. It should be remembered, however, that entrepreneurial discretion is the key to our new balance sheet. Sufficient subclassification should be done to allow for a full range of entrepreneurial discretion. For example, if several products are being manufactured, an account for the finished goods inventory of each product should be listed, instead of combining the total inventory into one account.

Non-variable assets are of more concern to us. Traditional balance sheets state the value of fixed assets in terms of dollars. This, however, is not the major concern of the entrepreneur as he plans his operations

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[1]It is true that some of the variable assets are committed to meet non-discretionary expenses such as taxes, guaranteed wage payments, etc., but the entrepreneur does have some latitude in the extent of utilization of variable assets.

for the next mono-periodic production period. He is more concerned with what can be done with the available assets than he is with the accountant's evaluation of them. There are some assets (perhaps the land of on which his factory operates, or even the factory building itself) which do not enter into the management decision process for the period under consideration. These assets might be omitted from our balance sheet without loss of effectiveness.

To better understand this concept of non-variable assets, let us introduce the concept of a *control unit* (center). Most managers, instead of viewing the factory as one big operation, cut the fabricating activity up into control centers. These control units can be one asset, or a group of assets working as a unit. The distinguishing characteristic of a control unit is that it operates as an entity. In regards to production, it represents an uninterrupted process flow. Any material which enters a control unit cannot be removed until it has undergone all operations and services offered by that control center. In a sense, there is no entrepreneurial discretion within a control center. Consider a tile manufacturing operation. If a firm has only one mixer, one press, and one furnace, then the firm may be thought of as having only one control center. Suppose, however, that the firm has two mixers, two presses, and two furnaces, and that the mixers may

feed either press, and each press may feed either of the two furnaces. The firm must now be thought of as having six control units.

Services under a guaranteed contract must also be considered as non-variable assets. If the firm has a contract guaranteeing a thirty hour workweek for some or all of its employees, then this must be considered as a non-variable asset. This amount of labor is available to the entrepreneur whether he uses it or not. Guarantees for other services such as a minimum amount of electricity or water should likewise be considered as non-variable assets.

The important consideration with non-variable assets is the amount available during the period. Consideration must be given to the fact that the units of measurement differ from asset to asset. The simplest method of handling these differences is to convert all availability to a percentage basis. Normally, 100% of each fixed asset will be available in each mono-periodic production period. There will be times, however, when less than 100% (a machine needs repair) of a non-variable asset is available.

The distinction between variable and non-variable liabilities should, as in the case of the assets, be made with respect to the entrepreneurial discretion involved.

Accrued wages, accounts payable, etc., are considered to be variable liabilities because they vary as the result of management decisions made about the operations of the period. The important consideration with variable liabilities is, as was the case with variable assets, to provide enough subclassification to insure entrepreneurial discretion.

Most firms, however, do have many items which can be considered as non-variable liabilities. The firm usually has a given tax bill to pay (or accrue) during each period. The same thing would hold true for salaries, guaranteed wages, and other items which the firm is committed to pay regardless of the operations during the mono-periodic production period.

Since some entrepreneurial decisions are based upon the effect of the chosen alternatives upon traditional balance sheet accounts, it is convenient to include these in the entrepreneurial discretion balance sheet. A typical entrepreneurial discretion balance sheet is shown in Table 3.

A Modified Concept of Processes

Let us next consider the concept of process (transaction). Our new concept of a process is almost identical to the previous one. Table 4 illustrates several transactions.

TABLE 3--HYPOTHETICAL ENTREPRENEURIAL BALANCE SHEET

Account Number	Account Name	Valuation
	Variable assets:	
101	Cash	20,000
102	Accounts receivable	100,000
	Inventory (finished goods)	
103	Product alpha	80,000
104	Product beta	20,000
	Inventory (raw materials)	
105	Material omega	45,000
106	Material zeta	15,000
		280,000
	Value of fixed assets:	
201	Land	50,000
202	Buildings	60,000
203	Equipment	62,000
		172,000
	Non-variable assets:	
301	Control unit A	100
302	Control unit B	100
303	Control unit C	100
304	Guaranteed labor	100
305	Other guarantees	100
	Variable liabilities:	
401	Accounts payable	50,000
402	Accrued wages	10,000
403	Other accruals	4,000
		64,000
	Long-term debt:	
501	Bonds payable	100,000
		100,000
	Stockholders' equity:	
601	Common stock	240,000
602	Retained earnings	48,000
		288,000
	Non-variable liabilities:	
701	Guaranteed wages	20,000
702	General and administrative	26,000
		46,000

Source: See Section X, *The Entrepreneurial Balance Sheet*, for source and explanation of data.

TABLE 4--HYPOTHETICAL ENTREPRENEURIAL PROCESS ILLUSTRATIONS

Account Number	Account Name	Process Number 1	2	3	4	5	6
101	Cash					100	-30
102	Accounts receivable				100	-100	
103	Alpha inventory	70	40	70	-100		
104	Beta inventory						
105	Omega inventory	-30	-20	-30			
106	Zeta inventory	-10	-20	-10			
201	Value of fixed assets						
301	Control unit A	- 1	- 2	- 1			
302	Control unit B	- 8	- 8	- 8			
303	Control unit C		- 1				
304	Guaranteed labor	- 2	- 2				
401	Accounts payable						
402	Accrued wages			25			-25
501	Value of long-term debt						
601	Stockholders' equity						
701	Guaranteed payments						

TABLE 4--*Continued*

Account Number	Account Name	Process Number 7	8	9	10	11
101	Cash	-30	-10	-200	-260	
102	Accounts receivable					
103	Alpha inventory					
104	Beta inventory					
105	Omega inventory	30				
106	Zeta inventory		10			
201	Value of fixed assets					-50
301	Control unit A					
302	Control unit B					
303	Control unit C					
304	Guaranteed labor					
401	Accounts payable					
402	Accrued wages					
501	Value of long-term debt					
601	Stockholders' equity					
701	Guaranteed payments			-200	-260	

Source: See Section X, *A Modified Concept of Processes*, for source and explanation of data.

Process 1 represents the production of one unit of product "alpha" by a particular method. This process produces \$70 of "alpha" inventory. It utilizes \$30 of "omega" inventory, \$10 of "zeta" inventory, 1% of the total capacity of cost unit A, 8% of the total capacity of cost unit B, and 2% of the labor which is under a guaranteed contract.[2] Notice that the accrued wages account is not affected. The firm has utilized a fixed asset (guaranteed labor) instead of acquiring a variable liability (wage payments).

Process 2 is another method of producing \$70 of "alpha" inventory. It utilizes different percentages of the firm's fixed assets than does process one. *Process 3* is identical to process one except that it does not utilize any of the pre-paid labor. Instead, it uses discretionary labor; labor which can be purchased or not purchased. It is obvious that process three will never be utilized as long as the entrepreneur is free to utilize more of process one.

Processes 4 through 8 are very similar to the traditional accounting transactions. *Process 4* represents

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[2]The assumption with non-variable assets (for example, cost unit A) is that there is 100% availability at the beginning of the period and that process one reduces (thus the minus sign) this availability by 1%.

the selling of a unit of "alpha" inventory on credit. *Process 5* represents the conversion of accounts receivable into cash. *Process 6* represents the action of paying wages. *Process 7* and *process 8* represent the purchasing (for cash) of "omega" and "zeta" inventories respectively.

Process 9 represents the payments of guaranteed wages and *process 10* represents the payments for general and administrative expenses used throughout the period. *Process 11* accounts for the depreciation of fixed assets during the period (time-period depreciation is assumed).

A Modified Form of Business Income

Let us assume that we have w accounts on our new balance sheet. Suppose we again assign a number j to each of the firm's accounts and assume that j takes on integer values from 1 to w. We will then group the accounts so that as j varies as shown below, the following type of accounts will be assigned the number represented by j.

Variable assets	$1 \leq j \leq p$
Value of fixed assets	$p + 1 \leq j \leq q$
Availability of fixed assets	$q + 1 \leq j \leq r$
Variable liabilities	$r + 1 \leq j \leq s$
Value of long-term debt	$s + 1 \leq j \leq t$
Valuation of owners' equity	$t + 1 \leq j \leq u$
Fixed liabilities	$u + 1 \leq j \leq w$

Let us also assume that there are n processes and we will let i again be the index of process number so that

$$1 \leq i \leq n.$$

Let x_i represent the level of utilization of process i, and x, the program vector, be

$$x = [x_1, x_2, \ldots, x_n].$$

a_{ji} will represent the net effect on the "j"th account by utilizing the process i at a unit level. Any process can then be represented by a process vector a_i,

$$a_i = [a_{1i}, a_{2i}, \ldots, a_{wi}].$$

The process matrix A will then be

$$A = (a_1, a_2, \ldots, a_n).$$

Suppose we define a new w component column vector d^4 so that

$$d^4 = [d_1^4, d_2^4, \ldots, d_w^4],$$

$$d_j^4 = 1, \qquad j = 1, \ldots, q,$$

$$d_j^4 = 0, \qquad j = q + 1, \ldots, r,$$

$$d_j^4 = -1, \qquad j = r + 1, \ldots, u,$$

$$d_j^4 = 0, \qquad j = u + 1, \ldots, w.$$

The accountant's version of net income is now represented by

$$(Ax)^T d^4 = z = \text{net income.}$$

Let $c = A^T d^4$,

$z = cx.$

This, however, is not the function which the entrepreneur would like to optimize. Suppose we define another w component column vector d^5 so that

$$d^5 = [d_1^5, d_2^5, \ldots, d_w^5],$$

$$\begin{aligned}
d_j^5 &= 1, & j &= 1, \ldots, p,\\
d_j^5 &= 0, & j &= p+1, \ldots, r,\\
d_j^5 &= -1, & j &= r+1, \ldots, s,\\
d_j^5 &= 0, & j &= s+1, \ldots, u,\\
d_j^5 &= -1, & j &= u+1, \ldots, w.
\end{aligned}$$

Now consider the function

$$(Ax)^T d^5 = \hat{z},$$

let $\hat{c}^T = A^T d^5,$

$\hat{z} = \hat{c}x.$

This is the function the entrepreneur would like to maximize. The reason it is preferred is that it either does not consider, or "washes out" actions which are forced upon the

entrepreneur and thus effectively only considers those actions which are associated with entrepreneurial discreation.

Let us again let y^1 be a w component column vector of balance sheet accounts at the beginning of the period and y^2 be a w component vector of balance sheet accounts at the end of the period.

$$y = Ax$$

represents the change in balance sheet accounts during the period.

SECTION XI

THE PORTRAYAL OF THE FIRM: A PARTIAL MATHEMATICAL FORMULATION

We have shown that

$$(Ax)^T d^5 = \hat{c}x = \hat{z}$$

is the objective function of the firm. Now let us briefly reconsider the entrepreneur's sphere of discretion. The constraints which define the entrepreneur's sphere of discretion will often be formulated from the entrepreneurial balance sheet. The balance sheet itself, however, is not per se a constraint or set of constraints. The constraints must be formed from the balance sheet.

We will often want to formulate a constraint in terms of an end of period balance sheet account, or in terms of net change in a balance sheet account during a period. Since y^1 is assumed to be known, and since the following relationships hold,

$$y^1 + y = y^2,$$

$$y_j^1 + y_j = y_j^2,$$

$$y = Ax,$$

$$y_j = \sum_{i=1}^{n} a_{ji} x_i,$$

it is irrelevant whether we express the constraints in terms of y components or y^1 components because constraints expressed in terms of one can always be converted to the other. For our purposes, it will normally be convenient to express constraints in terms of y components. Let us introduce a new vector e_k which will be defined so that

$$e_k x = \sum_{i=1}^{n} a_{ji} x_i .$$

Thus, $e_k = (a_{j1}, a_{j2}, \ldots, a_{jn})$,

$$e_k = (e_{k1}, e_{k2}, \ldots, e_{kn}).$$

There is no previously defined relationship between j and k. The relationship which does exist for any particular problem will be dependent only on the problem formulation.

There will also be occasions when we would like to formulate a constraint directly in terms of a process or processes instead of in terms of balance sheet accounts. (The balance sheet account constraints were indirectly formed in terms of processes.)

We may, for example, want to insure that process 1 is operated at less than or equal to a certain level, or that process 2 plus process 3 equals zero, etc. e_k for these constraints may be formed from these equations by letting the "i"th component of e_k equal the coefficient of the "i"th process in the constraint equation. For processes

not considered in the constraint equation, the corresponding component of e_k will be zero.

In general a constraint will be formulated so that some combination of processes is required to be less than, greater than, or equal to a predetermined constant. A typical constraint will take the form of

$$e_k x \; \{\leq, =, \geq\} b_k .$$

There are many possible types of constraint formulations for a typical firm. It would probably be unwise to spend a great amount of effort here on mathematical formulation of constraints. It would, however, be wise to select a few types of constraints which are common to most firms; show how these can be formulated mathematically, and show that the formulation corresponds to the form previously described. Survival constraints, minimum profit constraints, corporate image constraints, sales constraints, and direction production constraints are considered below.

Survival Constraints

Earlier, we noted that the firm has a compelling urge to survive. Furthermore a firm which maximizes profits might not survive because of inadequate liquidity, etc. Therefore many firms put financial constraints upon

their operations. These constraints often take the form of ratios among two or more types of asset and liability accounts. We will consider two typical ratios here, the current ratio and the liquidity ratio.

Current ratio constraint

One of the simplest balance sheet ratios used by a firm is known as the current ratio. It is a simple ratio of current assets to current liabilities. The current ratio is the most widely known analytical percentage in financial management. It is not too accurate, and it can be manipulated easily by management. The current ratio reports, in percentage, the shrinkage that can occur in conversion of current assets to cash before becoming unable to pay off current liabilities. It is often believed that the current ratio should be maintained at a value which is greater than or equal to some constant such as two. Let us define a current asset vector f^1, a current liability vector f^2, and a current ratio vector f^3. All will be w component column vectors and f^3 will equal $f^1 - 2f^2$, i.e.,

$$f^3 = f^1 - 2f^2.$$

The components of f^1 will vary as follows:

$$f_j^1 = 1, \qquad j = 1, \ldots, p,$$

$$f_j^1 = 0, \qquad j = p + 1, \ldots, w.$$

The components of f^2 will vary as follows:

$$f_j^2 = 0, \qquad j = 1, \ldots, r,$$

$$f_j^2 = 1, \qquad j = r + 1, \ldots, s,$$

$$f_j^2 = 0 \qquad j = s + 1, \ldots, w.$$

The current ratio constraint can be expressed as

$$CA/CL \geq 2,$$

$$CA - 2CL \geq 0,$$

$$CA = f^{1^T}(Ax) + \sum_{j=1}^{p} y_j^1,$$

$$CL = f^{2^T}(Ax) + \sum_{j=r+1}^{s} y_j^1,$$

$$CA - 2CL = f^{3^T}(Ax) + \sum_{j=1}^{p} y_j^1 - 2 \sum_{j=r+1}^{s} y_j^1 \geq 0,$$

$$\sum_{j=1}^{p} y_j^1 - 2 \sum_{j=r+1}^{s} y_j^1 = - K_1 \text{ (known constant)},$$

$$f^{3^T}(Ax) \geq K_1,$$

$$f^{3^T} \sum_{i=1}^{n} a_i x_i \geq K_1,$$

$$\sum_{i=1}^{n} f^{3^T} a_i x_i \geq K_1,$$

$$e_k x \geq K_1 = b_k,$$

$$e_k = (f^{3^T} a_1, f^{3^T} a_2, \ldots, f^{3^T} a_n).$$

Liquidity of assets constraint

Liquidity of assets is an important consideration in any firm and at times a firm might choose to make decisions so that its quick assets (cash and receivables) will be at least a certain (for example, 25%) percent of total assets. Suppose we let y_1^2 represent cash and y_2^2 represent receivables. f^4 will be the quick asset vector and f^5 will represent the total asset vector. f^6 will represent the liquidity ratio vector.

$$f^6 = f^4 - .25f^5.$$

The components of f^4 will vary as follows:

$$f_j^4 = 1, \qquad j = 1, 2,$$

$$f_j^4 = 0, \qquad j = 3, \ldots, w.$$

The components of f^5 will vary as follows:

$$f_j^5 = 1, \qquad j = 1, \ldots, q,$$

$$f_j^5 = 0, \qquad j = q + 1, \ldots, w.$$

The liquidity constraint can be formulated as follows:

$$QA \geq .25TA,$$

$$QA = f^{4^T}(Ax) + \sum_{j=1}^{2} y_j^1,$$

$$TA = f^{5^T}(Ax) + \sum_{j=1}^{q} y_j^1,$$

$$QA - .25TA = f^{6^T}(Ax) + \sum_{j=1}^{2} y_j^1 - .25 \sum_{j=1}^{q} y_j^1 \geq 0,$$

$$\sum_{j=1}^{2} y_j^1 - .25 \sum_{j=1}^{q} y_j^1 = - K_3 \text{ (known constant)},$$

$$QA \geq .25TA = f^{6^T}(Ax) \geq K_3,$$

$$f^{6^T} \sum_{i=1}^{n} a_i x_i \geq K_3,$$

$$\sum_{i=1}^{n} f^{6^T} a_i x_i \geq K_3,$$

$$e_k x \geq K_3 = b_k,$$

$$e_k = (f^{6^T} a_1, f^{6^T} a_2, \ldots, f^{6^T} a_n).$$

Satisfactory Profit Constraints

Earlier in this study we noted that the entrepreneur should always be aware of the value of the objective function to insure that other objectives do not take precedence over the making of a satisfactory profit. It was noted that a minimum profit constraint could be formulated, but the inclusion of this requirement into the set of constraints would probably be redundant. This is certainly true if the entrepreneur expresses his minimum profit requirement as an absolute value. There are cases, however, where it would be advisable to include in the set of constraints, a requirement for a minimum return on sales, or on operating assets.

Return on sales

Let t be the sales vector,

$$t = (t_1, t_2, \ldots, t_n).$$

t_i equals the value received for operating a selling process at a unit level. t_i will equal zero for all processes which are not selling processes.

$$\sum_{i=1}^{n} t_i x_i = \text{total sales},$$

$$\sum_{i=1}^{n} c_i x_i = \text{net income}.$$

Let us assume that a firm would like for net income to be greater than or equal to 5% of total sales.

$$NI \geq .05TS,$$

$$NI - .05TX \geq 0,$$

$$\sum_{i=1}^{n} c_i x_i - .05 \sum_{i=1}^{n} t_i x_i \geq 0,$$

$$\sum_{i=1}^{n} (c_i - .05t_i)x_i \geq 0,$$

$$e_k x \geq 0,$$

$$e_k = (\{c_1 - .05t_1\}, \ldots, \{c_n - .05t_n\}).$$

Return on operating assets

Next let us consider return of operating assets. Assume that the firm would like net income to be greater

than or equal to 1% of operating assets. Let f^7 be the operating assets vector, and d^4 as previously defined is the net income vector. f^8 will be the return on operating assets vector.

$$f^8 = d^4 - .01f^7,$$

$$f_j^7 = 1, \qquad j = 1, \ldots, q,$$

$$f_j^7 = 0, \qquad j = q + 1, \ldots, w,$$

$$f^{7^T}(Ax) + \sum_{j=1}^{q} y_j^1 = \text{operating assets},$$

$$d^{4^T}(Ax) = z = \text{net income},$$

$$NI \geq .01OA,$$

$$NI - .01OA \geq 0,$$

$$d^{4^T}(Ax) - .01f^7(Ax) = .01 \sum_{j=1}^{q} y_j^1 \geq 0,$$

$$.01 \sum_{j=1}^{q} y_j^1 = K_4 \text{ (known constant)},$$

$$d^{4^T}(Ax) - .01f^7(Ax) \geq K_4,$$

$$f^{8^T}(Ax) \geq K_4,$$

$$f^{8^T} \sum_{i=1}^{n} a_i x_i \geq K_4,$$

$$\sum_{i=1}^{n} f^{8^T} a_i x_i \geq K_4,$$

$$e_k x \geq K_4 = b_k,$$

$$e_k = (f^{8^T} a_1, f^{8^T} a_2, \ldots, f^{8^T} a_n).$$

Sales Constraints

Sales constraints are easily formulated mathematically. We will consider here three types of sales constraints which we shall call simple sales constraints, total sales constraints, and complementary product constraints. Simple sales constraints state that no more than a given amount of a certain product may be sold during the period under consideration. Since we have a process for each selling activity, the constraint will take the form that the corresponding process must be less than or equal to some predetermined constant.

$$x_t \leq K_6 \text{ (known constant)},$$

$$e_k x \leq K_6 = b_k,$$

$$e_{ki} = 1, \qquad i = t,$$

$$e_{ki} = 0, \qquad i \neq t.$$

Total sales constraints are extensions of simple sales constraints. A firm may be manufacturing three products and decide that total sales cannot exceed a given amount.

$$x_a + x_b + x_c \leq K_7 \text{ (known constant)},$$

$$e_k x \leq K_7 = b_k,$$

$$e_{ki} = 1, \qquad i = a, b, c,$$

$$e_{ki} = 0, \qquad i \neq a, b, c.$$

Complementary product constraints arise when a firm manufactures two products which must be sold in a given ratio. The classic example is left shoes and right shoes. For each left shoe sold there must be a right shoe sold. Suppose a firm must sell twice as much of one product as another.

$$2x_a = x_b,$$

$$2x_a - x_b = 0,$$

$$e_k x = 0,$$

$$e_{ki} = 2, \qquad i = a,$$

$$e_{ki} = -1, \qquad i = b,$$

$$e_{ki} = 0, \qquad i \neq a, b.$$

Direct Production Constraints

Direct production constraints are, by definition, constraints placed upon the value of accounts in the entrepreneurial balance sheet. As noted before, it is irrelevant whether the constraints are expressed in terms of y (net change) or y^2 (end of period) components because constraints expressed in terms of one can always be

converted to the other. In reality, most of the constraints concerning variable assets or variable liabilities will be first expressed in terms of y^2 components and then, for convenience, must be converted to terms of y components.

$$y_j^2 \leq K_8,$$

$$y_j^1 + y_j \leq K_8,$$

$$y_j \leq K_8 - y_j^1 = b_k,$$

$$\sum_{i=1}^{n} a_{ji}x_i \leq b_k,$$

$$e_k x \leq b_k,$$

$$e_k = (a_{j1}, a_{j2}, \ldots, a_{jn}).$$

Quite often, the variable assets and variable liabilities will be constrained in more than one way. For example, an inventory account may be constrained to be greater than a certain minimum value but less than a given maximum amount.

$$k_9 \leq y_j^2 \leq K_{10};$$

$$y_j^2 \geq K_9,$$

$$y_j \geq K_9 - y_j^1 = b_k,$$

$$e_k x \geq b_k;$$

$$y_j^2 \leq K_{10},$$

$$y_j \le K_{10} - y_j^1 = b_{k+1},$$

$$e_{k+1}x \le b_{k+1}.$$

Non-variable asset constraints are almost always expressed in terms of y components. To be more specific, the total amount of a non-variable asset utilized will usually be constrained to be less than 100%.

$$y_j \le 100,$$

$$\sum_{i=1}^{n} a_{ji}x_i \le 100 = b_k,$$

$$e_k x \le 100,$$

$$e_k = (a_{j1}, a_{j2}, \ldots, a_{jn}).$$

Non-variable liabilities are normally constrained to be zero (or a certain specific value) at the close of the period. In general, the total amount of any non-variable liability must be paid during the period so that the non-variable liability account on the entrepreneurial balance sheet must equal zero at the end of the period.

$$y_j^2 = 0,$$

$$y_j^1 + y_j = 0,$$

$$y_j = - y_j^1 = b_k,$$

$$\sum_{i=1}^{n} a_{ji}x_i = b_k,$$

$$e_k x = b_k,$$
$$e_k = (a_{j1}, a_{j2}, \ldots, a_{jn}).$$

Corporate Image Constraints

Earlier in this study we noted that the creation and maintenance of corporate images plays an important role in the modern firm. Ultimately, all decisions result from some sort of image in the mind of the decision-maker. The important consideration with corporate image constraints is what act, or what decisions, will project the correct corporate image to the group involved. These groups include customers, employees, competitors, stockholders, suppliers, government officials and the general public. Decisions result not from the actual images held by these groups, but by what acts management thinks will project the correct image.

The problem of image creation is complicated by the fact that the existence of a characteristic does not necessarily create an image of it, nor does an image insure the presence of the characteristics involved. Thus management is constrained by their belief in what will create the proper image, rather than by the image itself.

Depending upon the group involved, various types of images may be considered desirable by a firm. In most

cases, however, corporate image constraints will take the form of one of the other types of constraints presented.

Constraints which might project a desirable image to the general public would include the maintenance of a minimum total sales volume, maintenance of a minimum sales volume in some particular product line, or a minimum contribution to charitable and civic activities.

Stockholders may require a minimum return to invested capital. Government officials may frown on an excessive profit. Employees may be better satisfied if a certain level of contribution is made to a profit sharing program.

The corporate image constraints will vary greatly from firm to firm, but in general the formulation of the constraints will be similar to the formulation of other types of constraints. For this reason, no attempt will be made here to formulate typical corporate image constraints.

Review and Summary

The entrepreneur's sphere of discretion will be defined by a series of constraints of the form

$$e_k x\{\leq, \geq, =\}b_k.$$

Let us assume that there are m such constraints and that the entrepreneur's sphere of discretion is the

set of points which satisfies all m of the constraint equations. If all the constraint equations are converted to equality constraints, the entrepreneur's sphere of discretion can easily be converted to matrix form,

$$Ex = b.$$

The object of the restatement is to avoid some of the inequalities which resulted from economic analysis but which are awkward mathematically. This conversion is easily accomplished by the addition of slack and surplus processes. Each slack and surplus process vector will have a zero for each component except for the component corresponding to the process for which it was intended to convert from an inequality to an equality. This component will normally have a unit value. The exact interpretation of these slack and surplus processes will vary, but in general a slack process will represent the amount by which the program under consideration fails to satisfy the corresponding constraint as a strict equality, and a surplus process will represent the amount by which the program exceeds the minimum requirements of the corresponding constraint.

The firm is run by the entrepreneur. The entrepreneur is not one man, but a number of men who compose a management structure of groups and complexes; a

pluralistic whole. The entrepreneur, in carrying out the function of management, is primarily a decision-maker. An ultimate task of the entrepreneur is to select among alternatives which have varying opportunity costs. In this task, the entrepreneur will strive to select that alternative, or that set of alternatives which will optimize some predetermined objective. The entrepreneur is not free, however, to choose any set of alternatives. He must always select an alternative or set of alternatives which is within his sphere of discretion.

Mathematically, the entrepreneur would like to select a program, x, that will make his entrepreneurial discretion income, $\hat{z} = \hat{c}x$, as large as possible, subject to the limitations or constraints imposed. Succinctly, the problem is to find a program, x, which satisfies

$$Ex = b,$$

$$x \geq 0,$$

and which makes

$$\hat{z} = \hat{c}x$$

as large as possible.

SECTION XII

IMPLEMENTATION: THE PORTRAYAL OF THE FIRM

This section attempts to formulate, in terms of the concepts previously presented, the problem faced by one entrepreneur in a typical mono-periodic production period.

The Firm Chosen for the Implementing Study

A wall tile manufacturing plant was chosen for the implementation part of this study. There were two primary reasons for this choice. First, and perhaps most important, management was interested in the project and was extremely cooperative in providing the records, interviews, and other information from which the data presented in this section were obtained. Second, the organization of the firm and the types of problems faced by management serve well to illustrate the concepts previously presented.

The particular firm used for this implementation is one of the six largest firms in the ceramic tile manufacturing industry. It is a multiproduct, single-plant firm. The firm is run by a very progressive management team which has led the firm through a period

of rapid growth. For the past six years, the firm has experienced the most rapid growth in the industry.

Data Collection and Presentation

The data presented in this section were obtained from an extensive analysis of the firm's accounting, production and sales records, and from interviews with management personnel. The compilation of this data was one of the most difficult and time consuming aspects of this study.

Table 5 represents an actual entrepreneurial balance sheet for the firm at the beginning of a particular mono-periodic production period. Table 6 lists the various process alternatives available to the entrepreneur. Tables 7 through 11 mathematically describe (in vector form) the available processes. Table 12 lists the constraints which define the entrepreneur's sphere of discretion.

TABLE 5--BEGINNING OF PERIOD ENTREPRENEURIAL BALANCE SHEET

Account Number	Account Name	Valuation
	Variable assets:	
101	Cash	202,609
102	Accounts receivable	758,796
103	Prepaid expenses	21,653
	Inventory (finished goods)	
104	Bright 4-3/8	282,428
105	Bright trim	327,717
106	Matte 4-3/8	50,742
107	Matte trim	51,545
108	Dapple 4-1/4	43,506
109	Dapple trim	34,958
110	Crystal 4-3/8	36,635
111	Crystal trim	71,494
112	Hex 3	39,162
113	Hex 6	55,401
114	Scored ware	80,332
115	Scored trim	86,904
116	Sills	26,547
117	Six by Six	18,806
	Inventory (raw materials)	
118	Body material	108,123
119	Glaze material	208,832
120	Packing material	27,644
121	Mechanical stores	129,158
122	Die parts	49,839
123	Refractory supplies	60,010
	Value of fixed assets:	
201	Land	378,281
202	Building	2,177,846
203	Equipment	808,348
204	Other	44,386
	Total asset value	6,181,702
	Non-variable assets:	
301	Glaze preparation	420
302	Hand score	420
303	Straight-line number one	420
304	Straight-line number two	420

TABLE 5--*Continued*

Account Number	Account Name	Valuation
305	Machine score	420
306	Machine press	420
307	Straight-line number four	420
308	Straight-line number five	420
309	Spray	420
310	Hand spray	420
311	Kiln	420
312	Grading	420
	Variable liabilities:	
401	Notes payable	966,000
402	Accounts payable	464,949
403	Accrued salaries and wages	64,301
404	Accrued taxes	236,667
405	Miscellaneous accruals	51,410
	Long-term debt:	
501	Bonds payable	1,479,398
	Stockholders equity:	
601	Common stock	340,980
602	Excess over par	752,722
603	Retained earnings	1,825,275
	Total liabilities	6,181,702
	Non-variable liabilities:	
	Finance	
701	Dividends payable	-0-
702	Profit sharing contribution	6,800
703	Interest expense	22,761
704	Personal and property tax	8,865
705	Planned purchases	54,888
	General and administrative	
751	Sales division	21,398
752	Warehousing and distribution	18,629
753	Finance division	16,563
754	Executive division	16,990
755	Industrial relations	20,038
756	Expansion and diversification	5,446

TABLE 5--*Continued*

Account Number	Account Name	Valuation
757	Research and development	5,949
758	Maintenance	9,607
759	General factory expenses	23,519
760	Engineering supervision	7,056
761	Transportation division	16,527
762	Quality control	4,953
763	Engineering services	6,397

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

TABLE 6--KEY TO PROCESS NUMBERS

Process Number	Description
	Manufacturing:
1	Bright 4-3/8; method 1
2	Bright 4-3/8; method 2
3	Bright 4-3/8; method 3
4	Bright 4-3/8; method 4
5	Bright 4-3/8; method 5
6	Bright trim; method 1
7	Bright trim; method 2
8	Matte 4-3/8; method 1
9	Matte 4-3/8; method 2
10	Matte 4-3/8; method 3
11	Matte 4-3/8; method 4
12	Matte 4-3/8; method 5
13	Matte trim; method 1
14	Matte trim; method 2
15	Dapple 4-1/4; method 1
16	Dapple 4-1/4; method 2
17	Dapple 4-1/4; method 3
18	Dapple trim; method 1
19	Dapple trim; method 2
20	Crystal 4-3/8; method 1
21	Crystal 4-3/8; method 2
22	Crystal 4-3/8; method 3
23	Crystal 4-3/8; method 4
24	Crystal trim; method 1
25	Crystal trim; method 2
26	Hex 3; method 1
27	Hex 3; method 2
28	Hex 6; method 1
29	Hex 6; method 2
30	Scored ware
31	Scored trim; method 1
32	Scored trim; method 2
33	Sills
34	Six by six
	Selling:
40	Bright 4-3/8
41	Bright trim
42	Matte 4-3/8

TABLE 6--*Continued*

Process Number	Description
43	Matte trim
44	Dapple 4-1/4
45	Dapple trim
46	Crystal 4-3/8
47	Crystal trim
48	Hex 3
49	Hex 6
50	Scored ware
51	Scored trim
52	Sills
53	Six by six
	Purchasing:
54	Body material
55	Glaze material
56	Packing material
57	Mechanical stores
58	Die parts
59	Refractory supplies
	Finance:
60	Reduction of accounts receivable
61	Reduction of accounts payable
62	Reduction of notes payable
63	Payment of wages
64	Reduction of miscellaneous accruals
65	Transfer of long term debt
66	Dividend payments
67	Contribution to profit sharing
68	Interest expense
69	Interest earned
70	Income tax accruals
71	Income tax payments
72	Plant depreciation--buildings
73	Plant depreciation--equipment
74	Plant depreciation--other
75	Personal and property tax--accruals
76	Personal and property tax--payments
77	Machinery purchases
78	Charitable and civic

TABLE 6--*Continued*

Process Number	Description
	General and administrative:
80	Sales division
81	Warehousing and distribution
82	Finance division
83	Executive division
84	Industrial relations
85	Expansion and diversification
86	Research and development
87	Maintenance
88	General factory
89	Engineering supervision
90	Transportation
91	Quality control
92	Engineering services
93	Advertising

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

TABLE 7--MANUFACTURING PROCESSES

Account Number	Process Number 1	2	3	4	5
103					
104	250.000	250.000	250.000	250.000	250.000
105					
118	-70.800	-70.800	-70.800	-70.800	-70.800
119	-28.800	-28.800	-36.800	-36.800	-27.900
120	- 9.200	- 9.200	- 9.200	- 9.200	- 9.200
121	- 9.800	- 9.800	- 8.500	- 8.500	- 6.100
122	- 4.400	- 4.400	- 4.000	- 4.000	- 4.000
123	- 4.600	- 4.600	- 4.400	- 4.400	- 2.800
301	- .110	- .110	- .142	- .142	- .126
302					
303	- .776				
304		- .776			
305					
306					- .153
307			- 1.010		
308				- 1.010	
309					- .832
310					
311	- .155	- .155	- .155	- .155	- .155
312	- .108	- .108	- .124	- .124	- .108
402	10.000	10.000	13.300	13.300	17.200
403	48.300	48.300	61.200	61.200	75.800

TABLE 7--*Continued*

Account Number	Process Number 6	7	8	9	10
105	550.000	550.000			
106			276.000	276.000	276.000
107					
118	-79.700	-79.700	-70.800	-70.800	-70.000
119	-32.800	-32.800	-41.000	-41.000	-51.500
120	-15.500	-15.500	- 9.200	- 9.200	- 9.200
121	-16.400	-16.400	- 9.100	- 9.100	- 9.100
122	- 7.000	- 7.000	- 4.700	- 4.700	- 4.000
123	- 7.400	- 7.400	- 5.300	- 5.300	- 4.200
301	- 0.126	- 0.126	- 0.114	- 0.114	- 0.142
302					
303			- 0.776		
304				- 0.776	
305					
306	- 0.347	- 0.347			
307					- 1.010
308					
309	- 1.060				
310		-10.000			
311	- 0.212	- 0.212	- 0.155	- 0.155	- 0.155
312	- 0.265	- 0.255	- 0.111	- 0.111	- 0.128
402	14.400	14.400	10.300	10.300	13.600
403	127.600	137.600	49.000	49.000	62.100

TABLE 7--*Continued*

Account Number	Process Number 11	12	13	14	15
106	276.000	276.000			
107			567.000	567.000	
108					287.000
118	-70.800	-70.800	-79.200	-79.200	-63.800
119	-51.500	-35.700	-58.500	-58.500	-52.600
120	- 9.200	- 9.200	-15.500	-15.500	- 9.200
121	- 9.100	- 6.000	-16.500	-16.500	- 8.700
122	- 4.000	- 2.600	- 7.500	- 7.500	- 3.800
123	- 4.200	- 3.200	- 7.900	- 7.900	- 4.200
301	- 0.142	- 0.142	- 0.159	- 0.159	- 0.162
302					
303					
304					
305					
306		- 0.153	- 0.347	- 0.347	
307					- 0.875
308	- 1.010	- 2.000			
309			- 0.832	- 1.405	
310				-10.000	
311	- 0.155	- 0.155	- 0.212	- 0.212	- 0.155
312	- 0.128	- 0.111	- 0.265	- 0.265	- 0.124
402	13.600	17.400	14.700	15.200	13.400
403	62.100	85.700	140.800	146.300	59.500

TABLE 7--*Continued*

Account Number	Process Number 16	17	18	19	20
108	287.000	287.000			
109			572.000	572.000	
110					293.000
118	-63.800	-63.800	-79.200	-79.200	-70.800
119	-52.600	-45.700	-77.100	-77.100	-47.700
120	- 9.200	- 9.200	-15.500	-15.500	- 9.200
121	- 8.700	- 6.200	-16.000	-16.000	-11.200
122	- 3.800	- 2.600	- 7.500	- 7.500	- 4.500
123	- 4.200	- 2.700	- 7.800	- 7.800	- 5.000
301	- 0.162	- 0.236	- 0.236	- 0.236	- 0.139
302					
303					- 0.776
304					
305					
306		- 0.153	- 0.347	- 0.347	
307					
308	- 0.875				
309		- 0.832	- 0.832		
310				-10.000	
311	- 0.155	- 0.155	- 0.212	- 0.212	- 0.155
312	- 0.124	- 0.124	- 0.265	- 0.265	- 0.177
402	13.400	17.200	15.000	15.500	10.400
403	59.500	74.700	134.500	143.500	49.200

TABLE 7--*Continued*

Account Number	Process Number				
	21	22	23	24	25
110	293.000	293.000	293.000		
111				727.000	727.000
112					
118	-70.800	-70.800	-70.800	-79.200	-79.200
119	-47.700	-58.500	-58.500	-70.700	-70.700
120	- 9.200	- 9.200	- 9.200	-15.500	-15.500
121	-11.200	- 9.100	- 9.100	-17.000	-16.700
122	- 4.500	- 4.500	- 4.500	- 8.000	- 7.100
123	- 5.000	- 4.900	- 4.900	- 8.700	- 8.900
301	- 0.139	- 0.168	- 0.168	- 0.209	- 0.209
302					
303					
304	- 0.776				
305					
306				- 0.347	- 0.347
307		- 1.010			
308			- 1.010		
309				- 2.340	
310					-10.000
311	- 0.155	- 0.155	- 0.153	- 0.212	- 0.212
312	- 0.117	- 0.128	- 0.128	- 0.265	- 0.265
402	10.400	15.400	15.400	16.900	15.900
403	49.200	63.900	63.900	172.100	189.300

TABLE 7--*Continued*

Account Number	Process Number 26	27	28	29	30
112	417.000	417.000			
113			420.000	420.000	
114					420.000
118	-70.800	-70.800	-63.800	-63.800	-70.800
119	-70.700	-77.100	-77.100	-70.700	-27.900
120	-19.500	-19.500	- 8.500	- 8.500	- 9.200
121	-10.200	-23.400	-13.300	- 8.000	- 8.600
122	- 4.400	-10.000	- 5.800	- 3.800	- 3.200
123	- 4.600	-10.500	- 6.200	- 3.900	- 3.800
301	- 0.209	- 0.186	- 0.186	- 0.209	- 0.126
302					
303					
304					
305					- 1.470
306	- 0.218			- 0.218	- 0.153
307					
308		- 3.340	- 3.340		
309	- 1.115			- 1.115	
310					
311	- 0.271	- 0.271	- 0.174	- 0.174	- 0.156
312	- 0.196	- 0.196	- 0.221	- 0.212	- 0.126
402	26.300	33.300	20.500	34.800	26.400
403	109.700	152.300	106.100	125.900	383.100

TABLE 7--*Continued*

Account Number	Process Number			
	31	32	33	34
115	1,509.000	1,509.000		
116			498.000	
117				460.000
118	-79.700	-79.700	-77.000	-63.800
119	-32.800	-32.800	-32.800	-32.800
120	-15.500	-15.500	-15.500	-13.400
121	-30.100	-30.600	- 8.500	-10.700
122	-13.000	-13.300	- 3.900	- 4.300
123	-14.000	-14.000	- 4.500	- 4.700
301	- 0.126	- 0.126	- 0.126	- 0.126
302	- 4.760	- 4.760		
303				
304				
305				
306	- 0.347	- 0.347	- 0.494	- 0.494
307				
308				
309	- 1.060		- 1.050	- 0.861
310		-10.000		
311	- 0.212	- 0.212	- 0.205	- 0.269
312	- 0.265	- 0.265	- 0.149	- 0.159
402	24.60	24.60	35.80	38.60
403	334.60	337.60	125.20	123.80

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

TABLE 8--SELLING PROCESSES

Account Number	Process Number 40	41	42	43	44
101					
102	340.00	1,052.00	364.00	1,029.00	391.00
103					
104	-250.00				
105		-555.00			
106			-276.00		
107				-567.00	
108					-287.00
109					
110					
111					
112					
113					
114					
115					
116					
117					
401					
402	7.00	7.00	7.00	7.00	7.00
403	4.10	4.10	4.10	4.10	4.10
404					

TABLE 8--*Continued*

Account Number	Process Number 45	46	47	48	49
101					
102	1,385.00	457.00	1,211.00	609.00	539.00
103					
104					
105					
106					
107					
108					
109	-572.00				
110		-293.00			
111			-727.00		
112				-417.00	
113					-420.00
114					
115					
116					
117					
401					
402	7.00	7,00	7.00	7.00	7.00
403	4.10	4.10	4.10	4.10	4.10
404					

TABLE 8--*Continued*

Account Number	50	51	52	53
101				
102	531.00	1,630.00	794.00	760.00
103				
104				
105				
106				
107				
108				
109				
110				
111				
112				
113				
114	-420.00			
115		-1,509.00		
116			-498.00	
117				-460.00
401				
402	7.00	7.00	7.00	7.00
403	4.10	4.10	4.10	4.10
404				

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

TABLE 9--PURCHASING PROCESSES

Account Number	54	55	56	57	58	59
101						
102						
103						
118	1,000.00					
119		1,000.00				
120			1,000.00			
121				1,000.00		
122					1,000.00	
123						1,000.00
401						
402	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
403						

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

TABLE 10--FINANCE PROCESSES

Account Number	Process Number 60	61	62	63	64
101	1,000.00	-1,000.00	-1,000.00	-1,000.00	-1,000.00
102	-1,000.00				
103					
201					
202					
203					
204					
401			-1,000.00		
402		-1,000.00			
403				-1,000.00	
404					
405					-1,000.00
501					
701					
702					
703					
704					
705					

TABLE 10--*Continued*

Account Number	Process Number 65	66	67	68	69
101		-1,000.00			
102					1,000.00
103					
201					
202					
203					
204					
401	1,000.00				
402					
403					
404					
405			1,000.00	1,000.00	
501	-1,000.00				
701		-1,000.00			
702			-1,000.00		
703				-1,000.00	
704					
705					

TABLE 10--*Continued*

Account Number	Process Number 70	71	72	73	74
101		-1,000.00			
102					
103					
201					
202			-1,000.00		
203				-1,000.00	
204					-1,000.00
401					
402					
403					
404	1,000.00	-1,000.00			
405					
501					
701					
702					
703					
704					
705					

TABLE 10--*Continued*

Account Number	Process Number 75	76	77	78
101		-1,000.00		-1,000.00
102				
103				
201				
202				
203			1,000.00	
204				
401				
402			1,000.00	
403				
404	1,000.00	-1,000.00		
405				
501				
701				
702				
703				
704	-1,000.00			
705			-1,000.00	

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

TABLE 11--GENERAL AND ADMINISTRATIVE PROCESSES

Account Number	Process Number 80	81	82	83	84
201					
202					
203					
204					
401					
402	156.20	47.46	84.08	116.01	81.15
403	96.05	94.99	98.07	149.25	62.03
404					
405					
751	-252.25				
752		-142.45			
753			-182.15		
754				-265.26	
755					-143.18
756					
757					
758					
759					
760					
761					
762					
763					

TABLE 11--*Continued*

Account Number	Process Number				
	85	86	87	88	89
201					
202					
203					
204					
401					
402	5.77	18.53	11.00	119.54	
403	21.20	34.29	100.66	130.79	67.85
404					
405					
751					
752					
753					
754					
755					
756	-26.97				
757		-52.82			
758			-111.66		
759				-250.33	
760					-67.85
761					
762					
763					

TABLE 11--*Continued*

Account Number	90	91	92	93
201				
202				
203				
204				
401				
402	60.25	3.97	7.18	1,000.00
403	33.85	42.94	53.63	
404				
405				
751				
752				
753				
754				
755				
756				
757				
758				
759				
760				
761	-94.10			
762		-46.91		
763			-60.81	

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

TABLE 12--CONSTRAINTS DEFINING THE ENTREPRENEURIAL SPHERE OF DISCRETION

Constraint Number	Constraint Description
	Direct production constraints:
101	$\sum_{i=1}^{n} a_{101i}x_i \leq 47,391$
102	$-\sum_{i=1}^{n} a_{101i}x_i \leq 2,609$
103	$\sum_{i=1}^{n} a_{102i}x_i \leq 41,204$
104	$-\sum_{i=1}^{n} a_{102i}x_i \leq 58,796$
105	$\sum_{i=1}^{n} a_{103i}x_i \leq 20,347$
106	$-\sum_{i=1}^{n} a_{103i}x_i \leq 21,653$
107	$\sum_{i=1}^{n} a_{104i}x_i \leq 17,572$
108	$-\sum_{i=1}^{n} a_{104i}x_i \leq 32,428$
109	$\sum_{i=1}^{n} a_{105i}x_i \leq 2,283$

TABLE 12--*Continued*

Constraint Number	Constraint Description
110	$-\sum_{i=1}^{n} a_{105i}x_i \leq 27,717$
111	$\sum_{i=1}^{n} a_{106i}x_i \leq 1,258$
112	$-\sum_{i=1}^{n} a_{106i}x_i \leq 10,742$
113	$\sum_{i=1}^{n} a_{107i}x_i \leq 3,455$
114	$-\sum_{i=1}^{n} a_{107i}x_i \leq 11,545$
115	$\sum_{i=1}^{n} a_{108i}x_i \leq 1,494$
116	$-\sum_{i=1}^{n} a_{108i}x_i \leq 8,506$
117	$\sum_{i=1}^{n} a_{109i}x_i \leq 8,042$
118	$-\sum_{i=1}^{n} a_{109i}x_i \leq 958$
119	$\sum_{i=1}^{n} a_{110i}x_i \leq 365$

TABLE 12--*Continued*

Constraint Number	Constraint Description
120	$-\sum_{i=1}^{n} a_{110i}x_i \leq 2,958$
121	$\sum_{i=1}^{n} a_{111i}x_i \leq 506$
122	$-\sum_{i=1}^{n} a_{111i}x_i \leq 1,494$
123	$\sum_{i=1}^{n} a_{112i}x_i \leq 838$
124	$-\sum_{i=1}^{n} a_{112i}x_i \leq 9,162$
125	$\sum_{i=1}^{n} a_{113i}x_i \leq 4,599$
126	$-\sum_{i=1}^{n} a_{113i}x_i \leq 15,401$
127	$\sum_{i=1}^{n} a_{114i}x_i \leq 668$
128	$-\sum_{i=1}^{n} a_{114i}x_i \leq 5,332$
129	$\sum_{i=1}^{n} a_{115i}x_i \leq 3,096$

TABLE 12--*Continued*

Constraint Number	Constraint Description
130	$-\sum_{i=1}^{n} a_{115i}x_i \leq 1,904$
131	$\sum_{i=1}^{n} a_{116i}x_i \leq 453$
132	$-\sum_{i=1}^{n} a_{116i}x_i \leq 1,547$
133	$\sum_{i=1}^{n} a_{117i}x_i \leq 1,194$
134	$-\sum_{i=1}^{n} a_{117i}x_i \leq 806$
135	$\sum_{i=1}^{n} a_{118i}x_i \leq 1,877$
136	$-\sum_{i=1}^{n} a_{118i}x_i \leq 18,123$
137	$\sum_{i=1}^{n} a_{119i}x_i \leq 11,168$
138	$-\sum_{i=1}^{n} a_{119i}x_i \leq 28,832$
139	$\sum_{i=1}^{n} a_{120i}x_i \leq 2,356$

TABLE 12--*Continued*

Constraint Number	Constraint Description
140	$-\sum_{i=1}^{n} a_{120i}x_i \leq 644$
141	$\sum_{i=1}^{n} a_{121i}x_i \leq 25,842$
142	$-\sum_{i=1}^{n} a_{121i}x_i \leq 4,158$
143	$\sum_{i=1}^{n} a_{122i}x_i \leq 15,161$
144	$-\sum_{i=1}^{n} a_{122i}x_i \leq 4,839$
145	$\sum_{i=1}^{n} a_{123i}x_i \leq 10,990$
146	$-\sum_{i=1}^{n} a_{123i}x_i \leq 10$
147	$-\sum_{i=1}^{n} a_{301i}x_i \leq 420$
148	$-\sum_{i=1}^{n} a_{302i}x_i \leq 420$
149	$-\sum_{i=1}^{n} a_{303i}x_i \leq 420$

TABLE 12--*Continued*

Constraint Number	Constraint Description
150	$-\sum_{i=1}^{n} a_{304i}x_i \leq 420$
151	$-\sum_{i=1}^{n} a_{305i}x_i \leq 420$
152	$-\sum_{i=1}^{n} a_{306i}x_i \leq 420$
153	$-\sum_{i=1}^{n} a_{307i}x_i \leq 420$
154	$-\sum_{i=1}^{n} a_{308i}x_i \leq 420$
155	$-\sum_{i=1}^{n} a_{309i}x_i \leq 420$
156	$-\sum_{i=1}^{n} a_{310i}x_i \leq 420$
157	$-\sum_{i=1}^{n} a_{311i}x_i \leq 420$
158	$-\sum_{i=1}^{n} a_{312i}x_i \leq 420$
159	$\sum_{i=1}^{n} a_{401i}x_i \leq 34,000$

TABLE 12--*Continued*

Constraint Number	Constraint Description
160	$-\sum_{i=1}^{n} a_{401i}x_i \leq 66{,}000$
161	$\sum_{i=1}^{n} a_{402i}x_i \leq 35{,}051$
162	$-\sum_{i=1}^{n} a_{402i}x_i \leq 64{,}949$
163	$\sum_{i=1}^{n} a_{403i}x_i \leq 5{,}699$
164	$-\sum_{i=1}^{n} a_{403i}x_i \leq 44{,}301$
165	$\sum_{i=1}^{n} a_{404i}x_i \leq 63{,}333$
166	$-\sum_{i=1}^{n} a_{404i}x_i \leq 56{,}667$
167	$\sum_{i=1}^{n} a_{405i}x_i \leq 18{,}590$
168	$-\sum_{i=1}^{n} a_{405i}x_i \leq 31{,}410$
169	$-\sum_{i=1}^{n} a_{701i}x_i = 0$

TABLE 12--*Continued*

Constraint Number	Constraint Description
170	$-\sum_{i=1}^{n} a_{702i}x_i = 6,800$
171	$-\sum_{i=1}^{n} a_{703i}x_i = 22,761$
172	$-\sum_{i=1}^{n} a_{704i}x_i = 8,865$
173	$-\sum_{i=1}^{n} a_{705i}x_i = 54,888$
174	$-\sum_{i=1}^{n} a_{751i}x_i = 21,398$
175	$-\sum_{i=1}^{n} a_{752i}x_i = 18,629$
176	$-\sum_{i=1}^{n} a_{753i}x_i = 16,563$
177	$-\sum_{i=1}^{n} a_{754i}x_i = 16,990$
178	$-\sum_{i=1}^{n} a_{755i}x_i = 20,038$
179	$-\sum_{i=1}^{n} a_{756i}x_i = 5,446$

TABLE 12--*Continued*

Constraint Number	Constraint Description
180	$-\sum_{i=1}^{n} a_{757i}x_i = 5,949$
181	$-\sum_{i=1}^{n} a_{758i}x_i = 9,607$
182	$-\sum_{i=1}^{n} a_{759i}x_i = 23,519$
183	$-\sum_{i=1}^{n} a_{760i}x_i = 7,056$
184	$-\sum_{i=1}^{n} a_{761i}x_i = 16,527$
185	$-\sum_{i=1}^{n} a_{762i}x_i = 4,953$
186	$-\sum_{i=1}^{n} a_{763i}x_i = 6,397$
	Sales constraints:
201	$x_{40} \leq 1,000.0$
202	$.10x_{40} - x_{41} \leq 0$
203	$-.13x_{40} + x_{41} \leq 0$
204	$x_{42} \leq 53.0$

TABLE 12--*Continued*

Constraint Number	Constraint Description
205	$.10x_{42} - x_{43} \leq 0$
206	$-.15x_{42} + x_{43} \leq 0$
207	$x_{44} \leq 96.0$
208	$.09x_{44} - x_{45} \leq 0$
209	$-.12x_{44} + x_{45} \leq 0$
210	$x_{46} \leq 64.0$
211	$.14x_{46} - x_{47} \leq 0$
212	$.16x_{46} \div x_{47} \leq 0$
213	$x_{48} \leq 45.0$
214	$x_{49} \leq 25.5$
215	$x_{50} \leq 39.0$
216	$.08x_{50} - x_{51} \leq 0$
217	$-.10x_{50} + x_{51} \leq 0$
218	$x_{52} \leq 28.5$
219	$x_{53} \leq 2.0$

TABLE 12--*Continued*

Constraint Number	Constraint Description
	Survival constraints:
301	$\sum_{i=1}^{n} f^{3^T} a_i x_i \geq 97,851$
302	$\sum_{i=1}^{n} f^{6^T} a_i x_i \leq 618,170$
303	$\sum_{i=1}^{n} (c_i - .04t_i) \geq 50,000$
304	$\sum_{i=1}^{n} (f^{8^T} a_i x_i \geq 30,908$
	Corporate image constraints:
401	$x_{40} \geq 500.0$
402	$x_{42} \geq 40.0$
403	$x_{44} \geq 40.0$
404	$x_{46} \geq 40.0$
405	$x_{48} + x_{49} \geq 40.0$
406	$x_{50} \geq 20.0$
407	$x_{78} \geq 4.3$
408	$x_{93} \geq 14.0$

TABLE 12--*Continued*

Constraint Number	Constraint Description
	Marginal constraints:
501	$x_{62} = 0.0$
502	$x_{65} = 0.0$
503	$x_{66} = 0.0$
504	$x_{69} = 4.086$
505	$x_{70} = 0.0$
506	$x_{71} = 40.401$
507	$x_{72} = 38.937$
508	$x_{73} = 15.267$
509	$x_{74} = 0.66$
510	$x_{76} = 0.0$

Source: See Section XII, *Data Collection and Presentation*, for source and explanation of data.

SECTION XIII

IMPLEMENTATION: A COMPARISON OF AN ACTUAL AND AN OPTIMAL ENTREPRENEURIAL SOLUTION

Although this study is primarily concerned with a conceptual presentation rather than with any calculations which might result from the entrepreneurial model, it was thought that significance would be added to the conceptual presentation if an optimal solution was calculated and compared to the firm's actual operations during a particular mono-periodic production period.

The Two Solutions

The actual solution

Table 13 presents the actual operations of the firm during a particular mono-periodic production period. Table 14 presents the end of period entrepreneurial balance sheet which resulted from this actual solution. Before tax net income for the period is represented by the difference between total assets and total liabilities in the end of period entrepreneurial balance sheet. Actual before tax net income for this particular period amounted to $57,843.

TABLE 13--ACTUAL ENTREPRENEURIAL SOLUTION

Process	Value
1	532.641
2	330.029
3	227.370
4	0.000
5	0.000
6	125.301
7	0.000
8	0.000
9	10.840
10	11.980
11	0.000
12	0.000
13	12.452
14	0.000
15	43.260
16	30.790
17	0.000
18	21.574
19	0.000
20	0.000
21	7.780
22	28.536
23	13.654
24	9.977
25	0.000
26	32.862
27	0.000
28	0.000
29	0.000

TABLE 13--*Continued*

Process	Value
30	34.130
31	5.067
32	0.000
33	28.094
34	2.130
40	1,057.326
41	135.816
42	52.645
43	7.602
44	95.140
45	9.012
46	63.220
47	9.598
48	44.872
49	25.170
50	38.990
51	4.278
52	28.050
53	1.965
54	97.591
55	25.042
56	15.782
57	40.481
58	20.662
59	17.412
60	712.083
61	415.692
62	0.000
63	214.279

TABLE 13--*Continued*

Process	Value
64	13.576
65	0.000
66	0.000
67	6.680
68	22.761
69	4.086
70	0.000
71	40.401
72	38.937
73	15.267
74	0.660
75	8.865
76	0.000
77	54.888
78	4.300
80	84.827
81	131.220
82	90.930
83	64.050
84	139.950
85	201.920
86	112.623
87	86.037
88	93.951
89	104.000
90	175.633
91	105.579
92	105.200
93	14.000

Source: See Section XIII, *The actual solution*, for source and explanation of data.

TABLE 14--ACTUAL END OF PERIOD ENTREPRENEURIAL BALANCE SHEET

Account Number	Account Name	Valuation
	Variable assets:	
101	Cash	226,444
102	Accounts receivable	762,685
103	Prepaid expenses	21,653
	Inventory (finished goods)	
104	Bright 4-3/8	290,357
105	Bright trim	321,881
106	Matte 4-3/8	42,510
107	Matte trim	54,295
108	Dapple 4-1/4	37,453
109	Dapple trim	42,144
110	Crystal 4-3/8	32,753
111	Crystal trim	71,770
112	Hex 3	34,154
113	Hex 6	44,830
114	Scored ware	78,291
115	Scored trim	88,095
116	Sills	26,569
117	Six by six	18,882
	Inventory (raw materials)	
118	Body material	97,815
119	Glaze material	181,263
120	Packing material	27,949
121	Mechanical stores	154,120
122	Die parts	63,550
123	Refractory supplies	70,024
	Value of fixed assets	
201	Land	378,281
202	Buildings	378,281
203	Equipment	2,138,909
204	Other	43,726
	Total asset value:	6,198,372
	Non-variable assets:	
301	Glaze preparation	229
302	Hand score	396
303	Straight-line number one	7
304	Straight-line number two	149

TABLE 14--*Continued*

Account Number	Account Name	Valuation
305	Machine score	370
306	Machine press	332
307	Straight-line number four	113
308	Straight-line number five	13
309	Spray	148
310	Hand spray	420
311	Kiln	171
312	Grading	219
	Variable liabilities:	
401	Notes payable	966,000
402	Accounts payable	438,317
403	Accrued salaries	65,311
404	Accrued taxes	205,131
405	Miscellaneous accruals	67,395
	Long-term debt:	
501	Bonds payable	1,479,398
	Stockholders' equity:	
601	Common stock	340,980
602	Excess over par	752,722
603	Retained earnings	1,825,275
	Total liabilities:	6,140,529
	Non-variable liabilities:	
	Finance	
701	Dividends payable	0
702	Profit sharing contribution	0
703	Interest expense	0
704	Personal and property tax	0
705	Planned purchases	0
	General and administrative	
751	Sales division	0
752	Warehousing and distribution	0
753	Finance division	0
754	Executive division	0
755	Industrial relations	0
756	Expansion and diversification	0
757	Research and development	0

TABLE 14--*Continued*

Account Number	Account Name	Valuation
758	Maintenance	0
759	General factory	0
760	Engineering supervision	0
761	Transportation	0
762	Quality control	0
763	Engineering services	0

Source: See Section XIII, *The actual solution*, for source and explanation of data.

The optimal solution

The optimal solution was calculated by using a standard linear programming procedure. The problem was solved on an IBM 709 computer using the C-E-I-R LP/90 program. Table 15 presents the optimal solution. Table 16 presents the end of period entrepreneurial balance sheet which would have resulted if this optimal solution had actually been put into operation. Net income for this optimal program is $95,775. Before analyzing the differences in the two solutions, it would probably be wise to introduce another concept from the theory of linear programming. This concept is duality theory. Duality theory will help us analyze the differences between the actual and the optimal solutions.

Duality Theory

The dual problem

Given any linear programming problem (which may be referred to as the primal problem):

$$\max z = cx,$$
$$\text{s.t. } Ex \leq b,$$
$$x \geq 0,$$

there is another linear programming problem, called the dual, which can be represented as:

TABLE 15--OPTIMAL ENTREPRENEURIAL SOLUTION

Process	Value
1	418.4333
2	541.237
3	210.617
4	0.000
5	0.000
6	147.113
7	0.000
8	57.558
9	0.000
10	0.000
11	0.000
12	0.000
13	14.043
14	0.000
15	101.206
16	0.000
17	0.000
18	25.579
19	0.000
20	65.246
21	0.000
22	0.000
23	0.000
24	10.936
25	0.000
26	47.009
27	0.000
28	36.450
29	0.000

TABLE 15--*Continued*

Process	Value
30	26.305
31	5.952
32	0.000
33	29.410
34	4.596
40	1,100.000
41	143.000
42	53.000
43	7.950
44	96.000
45	11.520
46	64.000
47	10.240
48	45.000
49	25.500
50	39.000
51	3.900
52	28.500
53	2.000
54	106.162
55	33.923
56	17.311
57	44.084
58	23.303
59	8.661
60	800.549
61	473.756
62	0.000
63	223.730
64	60.971

TABLE 15--*Continued*

Process	Value
65	0.000
66	0.000
67	6.800
68	22.761
69	4.086
70	0.000
71	40.401
72	38.937
73	15.267
74	0.660
75	8.865
76	0.000
77	54.888
78	4.300
80	84.828
81	131.217
82	90.930
83	64.050
84	139.950
85	201.928
86	112.628
87	86.038
88	93.951
89	103.994
90	175.632
91	105.585
92	105.196
93	14.000

Source: See Section XIII, *The optimal solution*, for source and explanation of data.

TABLE 16--OPTIMAL END OF PERIOD ENTREPRENEURIAL BALANCE SHEET

Account Number	Account Name	Valuation
	Variable assets:	
101	Cash	200,000
102	Accounts receivable	701,743
103	Prepaid expenses	21,653
	Inventory (finished goods)	
104	Bright	300,000
105	Bright trim	330,000
106	Matte 4-3/8	52,000
107	Matte trim	55,000
108	Dapple 4-1/4	45,000
109	Dapple trim	43,000
110	Crystal 4-3/8	37,000
111	Crystal trim	72,000
112	Hex 3	40,000
113	Hex 6	60,000
114	Scored ware	75,000
115	Scored trim	90,000
116	Sills	27,000
117	Six by six	20,000
	Inventory (raw materials)	
118	Body material	90,000
119	Glaze material	180,000
120	Packing material	27,000
121	Mechanical stores	155,000
122	Die parts	65,000
123	Refractory supplies	60,000
	Value of fixed assets:	
201	Land	378,281
202	Buildings	2,138,909
203	Equipment	847,969
204	Other	43,726
	Total asset value:	6,155,281
	Non-variable assets:	
301	Glaze preparation	198
302	Hand score	392
303	Straight-line number one	0
304	Straight-line number two	0

TABLE 16--*Continued*

Account Number	Account Name	Valuation
305	Machine score	381
306	Machine press	318
307	Straight-line number four	119
308	Straight-line number five	365
309	Spray	95
310	Hand spray	420
311	Kiln	130
312	Grading	184
	Variable liabilities:	
401	Notes payable	966,000
402	Accounts payable	400,000
403	Accrued salaries	70,000
404	Accrued taxes	205,131
405	Miscellaneous accruals	20,000
	Long-term debt:	
501	Bonds payable	1,479,398
	Stockholders' equity:	
601	Common stock	340,980
602	Excess over par	752,722
603	Retained earnings	1,825,275
	Total liabilities:	6,059,506
	Non-variable liabilities:	
	Finance	
701	Dividends payable	0
702	Profit sharing contribution	0
703	Interest expense	0
704	Personal and property tax	0
705	Planned purchases	0
	General and administrative	
751	Sales division	0
752	Warehousing and distribution	0
753	Finance division	0
754	Executive division	0
755	Industrial relations	0
756	Expansion and diversification	0
757	Research and development	0

TABLE 16--*Continued*

Account Number	Account Name	Valuation
758	Maintenance	0
759	General factory	0
760	Engineering supervision	0
761	Transportation	0
762	Quality control	0
763	Engineering services	0

Source: See Section XIII, *The optimal solution,* for source and explanation of data.

$$\min Z = b^T w,$$
$$\text{s.t. } E^T w \geq c^T,$$
$$w \geq 0.$$

Primal-dual relationships

There are several interesting relationships between the primal and the dual problems. These relationships are given below and can easily be proved (Hadley, 1962, pp. 224-242).

1. If the primal problem has an optimal solution, then the dual also has an optimal solution.

2. If $\hat{x}$ is a feasible solution to the primal, and $\hat{w}$ is a feasible solution to the dual such that $cx = b^T w$, then $\hat{x}$ is an optimal solution to the primal and $\hat{w}$ is an optimal solution to the dual.

3. If a slack or surplus variable x_{n+j}, which has been added to the "j" primal constraint, appears in an optimal basic solution, then for the corresponding optimal solution to the dual, the "j"th dual variable is zero, that is $w_j = 0$.

4. If the variable x_i appears in an optimal basic solution to the primal problem, then in the corresponding optimal solution to the dual, the "i"th dual constraint holds as a strict equality, that is, the dual slack or surplus variable $w_{m+i} = 0$.

Economic interpretation of the dual variables

First let us consider the physical dimensions of the variable in our problem. The dimensions of the variables x_i will be units of some process operated per period. The dimensions of the b_j vary depending upon the

concerned constraint, but, in general, they represent units of resource j available in a given mono-periodic production period. The a_{ji} have the dimensions of units of resource j per unit of process i. The dimensions of c_i are dollars per unit of process i. $a_{ji}w_j$ must have the dimensions of dollars per unit of process i. But since the dimensions of a_{ji} are units of resource j per unit of process i, it must be true that the w_j have the dimensions of dollars per unit of resource j.

To each resource j there corresponds a dual variable w_j which, by its dimensions, is a value to be associated with one unit of resource j. The dual variables are sometimes referred to as shadow prices or imputed values of the resources. This valuation is an opportunity cost valuation. If it were possible to increase or decrease the amount of available resource j by one unit without changing the dual solution, then profit would be increased or decreased by w_j. Of course, if b_j were actually changed to $b_j + 1$, the profit would not, in general, increase by w_j because the entire optimal solution changes. Nevertheless, w_j can be interpreted as the $\partial z/\partial b_j$. Thus when z is being maximized, w_j is a measure of the rate of change of z with respect to b_j. If any resource is not fully utilized in an optimal solution, z will not change

if the availability of the resource is changed slightly. Thus, the dual variable associated with this resource should be zero.

Table 17 lists the values of the dual variables associated with the constraints defining the entrepreneurial sphere of discretion. The value of the dual variable is zero for each associated constraint which is not satisfied as a strict equality. Dual variables with a zero valuation have been omitted from Table 17. In order to better illustrate this duality concept, let us choose some typical constraints and illustrate more completely the significance of the dual variables.

Illustrations of dual variables

Direct production constraints.--Constraints 1 through 186 are direct production constraints. The dimensions of b_j, $j = 147, \ldots, 158$, are percent of weekly capacity available. Consider constraint 149. The associated dual variable has a value of 28.7371. Thus a 1% increase in weekly capacity should yield an increase in net income of $28.74. An increase of 1% would allow the utilization of process 1 to increase by 1.29 (This allows 1.29 additional units of bright wall tile to be run on straight-line number one, which is the

TABLE 17--OPTIMAL DUAL VARIABLES ASSOCIATED WITH CONSTRAINTS DEFINING THE ENTREPRENEURIAL SPHERE OF DISCRETION

Constraint	Dual Variable Valuation
107	0.1672
109	0.4580
111	0.1967
113	0.3993
115	0.2500
117	0.3836
119	0.2140
121	0.4662
123	0.2417
125	0.2826
128	0.2691
129	0.6393
131	0.3922
133	0.3650
149	28.7371
150	28.7371
201	216.9130
203	740.1000
204	232.7950
206	677.3000
207	287.2560
209	1,021.3000
210	345.4800
212	811.8000
213	281.7000
214	226.6600
215	94.3600
217	1,074.6000
218	480.2000
219	456.8000

Source: See Section XIII, *Economic interpretation of the dual variables*, for source and explanation of data.

more efficient straight-line.) which provides for a net contribution[1] to profit of \$82.86. 1.29 less units of process 3 (production of bright wall tile on straight-line number four, the less efficient straight-line) would therefore be utilized. This would decrease net income by \$54.12. The marginal value of this resource and hence the value of the dual variable is \$82.86 minus \$54.12 which is \$28.74.

The dimensions of b_j, $j = 1, \ldots, 146$, and $j = 159, \ldots, 186$ are net change in dollar valuation of balance sheet accounts. Notice that the associated dual variables are all zero except for those associated with the final goods inventory constraints. Consider constraint 111, which is associated with the maximum amount of matte wall tile inventory in terms of dollar valuation. The associated dual variable has a value of 0.1967 which indicates that an increase in the inventory limit of \$1.00 would add \$0.20 of net income to the firm. This is easily verified. A unit increase in b_{111} would allow an additional 0.00362 units of matte wall tile to be produced. These units would be produced on straight-line number one (process 8). However since straight-line

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[1] The per unit contribution of any process can be obtained by $c = A^T d^4$. For convenience, the net contribution of the manufacturing and selling processes are given in Table 18.

TABLE 18--NET CONTRIBUTION VALUES ASSOCIATED WITH MANUFACTURING AND SELLING PROCESSES

Process	Net Contribution Values
1	64.10
2	64.10
3	41.80
4	41.80
5	37.70
6	254.20
7	244.20
8	76.60
9	76.60
10	51.50
11	51.50
12	45.40
13	226.40
14	220.80
15	71.80
16	71.80
17	65.40
18	219.40
19	209.40
20	85.00
21	85.00
22	56.70
23	56.70
24	338.90
25	323.40
26	100.80

TABLE 18--*Continued*

Process	Net Contribution Values
27	20.10
28	118.70
29	100.60
30	-113.00
31	964.70
32	961.20
33	195.30
34	167.90
40	78.90
41	485.90
42	76.90
43	450.90
44	92.90
45	801.90
46	152.90
47	472.90
48	180.90
49	107.90
50	99.90
51	109.90
52	284.90
53	284.90

Source: See Section XIII, *Illustrations of dual variables*, for source and explanation of data.

number one is presently being utilized at full capacity, 0.00362 units of bright wall tile must be shifted from straight-line number one (process 1) to straight-line number four (process 3). Thus the dual variable should be 0.00362 times the sum of the net contributions of process 3 and process 8 minus the net contribution of process 1;

$$w_{111} = .00362(76.60 + 41.80 - 64.10) = 0.1967.$$

This procedure points to one of the big disadvantages of a mono-periodic production period model. The greater the volume of any product manufactured, the less will be the per unit overhead associated with the product. Thus, the model will believe it can create income by utilizing fixed capacity. The optimal solution will therefore contain as much inventory as the constraints will allow. In our problem, the limiting constraints are the inventory constraints. In other problems it could be a different constraint such as a limit on production facilities. In reality, this disadvantage of the model is not so great. If the problem is formulated correctly, it is reasonable to expect the entrepreneur to utilize as much of his fixed assets as possible, and therefore prepare for future mono-periodic production periods where excess capacity might not be available.

It is interesting to note that the dual variables associated with non-variable liability constraints are zero. It is certainly true that an increase or decrease in these constraints would change the net income for the period. Non-variable liabilities, however, are beyond the control of entrepreneurial discretion. The corresponding dual variables have a zero valuation. This is as it should be.

Sales constraints.--Sales constraints are of two types; those which set limits on the amount of primary product which can be sold, and those which limit the ratios of primary products sold to trim sold. Consider constraint 204. The associated dual variable has a value of 232.7950. If b_{204} is increased by one unit, this will allow process 42 and process 8 to be increased by one unit, and will allow process 43 and process 13 to be increased by 0.15 units.

In order to allow process 8 to be increased by one unit, one unit of bright wall tile must be shifted from straight-line number one to straight-line number four (process 1 to process 3). Thus,

$$w_{204} = c_{42} + c_8 + .15(c_{43} + c_{13}) - c_1 + c_3,$$

$$w_{204} = 76.90 + 76.60 + .15(450.90 + 226.40)$$
$$- 64.10 + 41.80,$$

$$w_{204} = 232.795.$$

Now consider constraint 203 which limits the ratio of bright trim sold to bright wall tile sold. If b_{203} is increased by one unit, this will allow process 41 and process 6 to increase by one unit and therefore add $740.10 to net income. $740.10 is, of course, the value of the dual variable.

Remaining constraints.--It should be noted that the value of the dual variables associated with all the remaining constraints are zero. Thus, nothing can be gained by changing the corresponding b value by an incremental unit.

Comparison of the Actual and the Optimal Solutions

Before comparing the optimal and actual solutions, two facts should be noted. Our actual aolution violates constraint 217 by 0.379 units, thus indicating a slight problem misformulation. However the discrepancy is not considered to be serious and therefore is ignored. Except for this one exception in the actual solution, both solutions satisfy all constraints. The firm's actions are therefore dominated by the highest order goal in the firm's hierarchy of goals. Thus, the firm's actions are dominated by the profit motive. Second, we should note that there are alternative optima in the optimal solution. For example, if process 60 (reduction of accounts receivable)

had been operated at a slightly lower level, all constraints would still be satisfied, and the value of the objective function would not have changed.

An entrepreneur, presented with an optimal solution, would probably question what accounted for the difference in net income between the actual solution and the optimal solution. This is the approach we shall take in comparing the two solutions.

To aid us in our comparison we shall utilize the value of the dual variables. Earlier, however, we noted that if b_j is actually changed by one unit, z might not actually change by w_j because the whole optimal solution may change. Therefore in utilizing the dual variables, we must insure that we only utilize them over an appropriate range. For this reason, we have broken the analysis into four subproblems.

Subproblem I

Subproblem I is concerned primarily with efficient production scheduling. In subproblem I, all processes, except production processes, are constrained to have the same value as the corresponding value in the actual solution. Furthermore, the total amount of each product manufactured is constrained to equal the amount manufactured in the actual solution. In effect, this problem

minimizes the cost of manufacturing the output of the actual solution. A summary of the results is given in Table 19.

The solution of subproblem I tells management to utilize the more efficient straight-lines to the fullest possible extent. The amount of bright wall which cannot be made on the more efficient straight-lines (because of capacity limitations) should be made on straight-line number three. The total savings from this shifting of production amounts to $4,773.

Subproblem II

Subproblem II is concerned with ratio sales constraints. Management felt that they could sell a volume of trim which was equal to or less than a given percentage of primary product. Except for the case previously mentioned, the failure to satisfy these constraints as strict equalities cost the firm money. A summary of the profit losses is given in Table 20. The total profit loss amounts to $3,881.

Subproblem III

Subproblem III is concerned with the profit loss which resulted from the failure to sell as much primary product as management felt it was possible to sell. It should be remembered that the value of the dual variables

TABLE 19--PROFIT LOSSES IN SUBPROBLEM I

Constraint	Slack	Dual Value	Profit Loss
SP1	147.014	22.30	$3,278
SP2	11.980	25.10	301
SP3	42.190	28.30	1,194
		Total Profit Loss	$4,773

Source: See Section XIII, *Subproblem I*, for source and explanation of data.

TABLE 20--PROFIT LOSSES IN SUBPROBLEM II

Constraint	Slack	Dual Value	Profit Loss
203	1.636	740.100	$1,211
206	0.296	677.700	201
209	2.405	1,021.300	2,456
212	0.517	811.800	420
217	-0.379	1,074.600	-407
		Total Profit Loss	$3,881

Source: See Section XIII, *Subproblem II*, for source and explanation of data.

takes into account all related changes which must be made. It therefore considers such things as production changes which must be made, and the effect upon sales of complementary products. A summary of the profit losses is given in Table 21. The total profit loss amounts to $10,201.

Subproblem IV

Subproblem IV considers the net income loss which results from the failure to satisfy inventory constraints as strict equalities. As discussed before, the question of whether this is an actual income loss is highly questionable. Nevertheless, if the problem has been formulated correctly, it must be assumed that there is some value in utilizing the excess capacity. A summary of the profit losses is given in Table 22. Total profit loss amounts to $19,077.

Summary of subproblems

The sum of the profit losses from subproblems I through IV amounts to $37,932. When this value is added to $57,843, which is the net income for the actual solution, the result is $95,775 which is the net income for the optimal solution.

TABLE 21--PROFIT LOSSES IN SUBPROBLEM III

Constraint	Slack	Dual Value	Profit Loss
201	42.674	216.913	$ 9,257
204	0.355	232.795	83
207	0.860	287.256	247
210	0.780	345.480	269
213	0.128	281.700	36
214	0.330	226.600	75
215	0.010	94.360	2
218	0.450	480.200	216
219	0.035	456.800	16
		Total Profit Loss	$10,201

Source: See Section XIII, *Subproblem III*, for source and explanation of data.

TABLE 22--PROFIT LOSSES IN SUBPROBLEM IV

Constraint	Slack	Dual Value	Profit Loss
107	9,643	0.1672	$ 1,612
109	8,119	0.4580	3,719
111	9,490	0.1967	1,867
113	705	0.3993	282
115	7,547	0.2500	1,887
117	856	0.3836	328
119	4,247	0.2140	909
121	230	0.4662	107
123	5,846	0.2417	1,413
125	15,170	0.2826	4,272
128	3,291	0.2691	886
129	1,905	0.6393	1,218
131	431	0.3992	169
133	1,118	0.3650	408
		Total Profit Loss	$19,077

Source: See Section XIII, *Subproblem IV*, for source and explanation of data.

Economic Activity in the Mono-periodic Period Chosen for Analysis

Monetary conditions in the United States during the particular mono-periodic production period chosen for analysis could be described by the phrase, "tight money." Interests rates on mortgages were high. This might account for the fact that demand for houses and building materials was low. Our firm, therefore, was operating at considerably below capacity. This fact made it much easier for us to analyze the differences between the optimal and actual solutions. In a period of high economic activity, the problem would have been much more difficult and would have required more subproblems.

This type of analysis, however, would be much more valuable to a firm if the analysis were made on a period in which there was a high rate of economic activity. In our problem, the dual variables were easy to calculate and illustrate. In fact, the value of the dual variables probably could have been obtained without any type of formal programming technique. In a period of high economic activity, the number of dependent relationships would be greatly increased, and the value of the dual variables would be much more difficult to estimate.

The model presented could have been formulated in much more detail. Both the number of processes and the

number of balance sheet components could have been increased. More constraints could have been imposed, and more dependent relationships between the process could have been formulated. Such an extension of the model might be valuable. For our purposes, however, it seemed as though more could be gained by moderate aggregation. For one reason, the concepts could be illustrated better.

The Opportunity Costs of Excluded Alternatives

Since we have introduced the concept of dual variables, we should probably point out that another interpretation of them is that they are the negative of the opportunity costs of the excluded alternatives. In Section VI, when we were discussing the entrepreneurial process, we noted that the entrepreneur should always choose that alternative with the highest opportunity cost. Furthermore, we said that if an entrepreneur ever reaches a point in the entrepreneurial process where the opportunity costs of all excluded alternatives are non-positive, then he can do no better. Table 23 lists the dual variables in a slightly different form than Table 17. In Table 23, the dual variables are associated with the excluded alternatives. All dual variables are non-negative therefore all opportunity costs are non-positive. Consider the dual

TABLE 23--OPTIMAL DUAL VARIABLES ASSOCIATED WITH PROCESSES OMITTED FROM THE OPTIMAL ENTREPRENEURIAL SOLUTION

Process	Dual Variable Valuation
4	0.0000
5	4.1000
7	10.0000
9	0.0000
10	2.8000
11	2.8000
12	8.9000
14	5.6000
16	0.0000
17	6.4000
19	10.0000
21	0.0000
22	6.0000
23	6.0000
25	15.5000
27	80.7000
29	18.1000
32	3.5000

Source: See Section XIII, *The Opportunity Costs of Excluded Alternatives*, for source and explanation of data.

variable associated with process 10. If this process were operated at a unit level, net income would be increased by $51.50. However one unit of process 8 must be removed at a cost of $76.60. This would allow process 1 to be increased by one unit with a resulting net contribution of $64.10. One unit of process 3 must then be removed at a cost of $41.80. The net result is a cost of $2.80. The values of the dual variables given in Table 13 are the negative of the opportunity costs associated with the slack or surplus variables which were originally introduced to convert all inequalities constraints to equality constraints.

The opportunity costs of some excluded alternatives are zero. Thus, at least an incremental unit of these processes could be utilized without changing the value of the optimal solution. This simply points to the fact that we do have alternative optima in the optimal solution.

SECTION XIV

COMMENTS ON THE PRESENTATION

The major work of this study is complete. Perhaps, however, we should briefly review the purpose of the presentation of the model and then show that the model presented may serve as a basis for the formulation of more efficient and more operational models.

The Model Presented

It is obvious that the entrepreneurial model presented is not an efficient problem-solving model. The intent was not to develop an efficient mathematical model, but rather to set the portrayal of the entire firm into a programming framework. Mathematicians and operations researchers have made rapid progress in using the tools of quantitative analysis to aid management in solving particular problems of the firm. It is significant, however, that management theorists have made less use of the inherent qualities of mathematical programming to advance the theory of the firm. Koontz and O'Donnell (1959, p. 527) state the following about the methods of operations research.

. . . operations research is essentially the application of scientific method to problems. As such, it is not new, since attempts have long been made to approach business and management problems scientifically. But what does have an element of novelty, is the orderliness and completeness of the approach to business problems which operations researchers have attempted to develop. They have placed emphasis on defining the problem and the goals, on carefully collecting and evaluating the facts, on developing and testing hypotheses from the myriad of facts bearing on a problem, on determining relationships between these facts, on developing and checking predictions based on these hypotheses, and on devising measures by which the effectiveness of a course of action leading toward a goal can be evaluated.

Thus the essential methods of operations research may be summarized as:

1. The emphasis on goals in a problem area and the development of effectiveness in determining whether a solution shows promise of attaining the goal. For example, if the goal is profit, the measure of effectiveness may be the rate of return on investment, and every proposed solution will arrange the variables and parameters involved so that the end result can be weighed against this measure.

2. The attempt to incorporate all the parameters bearing on a problem, or at least those which appear to be important to its solution, in an analysis.

3. The emphasis on models--the logical representation of a problem. These may, of course, be simple or complex.

4. The attempt to quantify the parameters in a problem to the extent possible, since only quantifiable data can be inserted into a model to yield a finite result or value for prediction.

5. The attempt to supplement quantifiable data with such usable mathematical and statistical limits as the probabilities in a situation, thus making the mathematical and computing problem workable within a small, and relatively insignificant, margin of error.

These features should be valuable to the management theorist in his attempt to improve the portrayal of the firm. It is primarily in this light that this study is offered.

Modifications of the Model Presented

We have noted that this study was not primarily concerned with efficient mathematical models which would aid the entrepreneur in obtaining solutions to particular problems. Nevertheless, it would be worthwhile briefly to show how the entrepreneurial model presented could be used as the basis for deriving mathematical models which would be of operational value to the firm. A brief discussion of model modifications is given below.

Profitability of product lines

The profitabilities of various product lines were obtained from our simple entrepreneurial model. The profitability factors of product lines in the implementing study are represented by the dual variable values in subproblem III, Section XIII.

These data are obviously valuable, on a routine basis, for such purposes as aiding sales personnel to determine the degree of emphasis to be placed upon various product lines. They are also valuable for special

programs, such as determining if a price concession[1] can be made for an exceptional, very large order.

The value of the model would be greatly enhanced if it were expanded to distinguish among product line sales by geographic location, territory, or distributor. Let us consider a model which is expanded to include product line sales by territory. The dual problem then indicates the profitability factors of product lines in each territory. The expanded model would include a selling process for each product in each territory. Additional sales constraints would also have to be formulated, but the value of the increase in information should far outweigh the cost of the model expansion.

Consider the case where a distributor asks for a special price for a very large order. The inclusion of a process for this potential order into the expanded model would allow the firm to determine not only the total profitability of the order, but also the profitability of this process at various levels of utilization. In making such a determination, the firm's analyst would have to give careful consideration to the value of the associated dual variable, and to the ranges over which a

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[1]This does not imply that the firm chosen for the implementing study maintained a policy of granting price concessions in exceptional cases. This example is chosen for illustrative purposes only.

particular dual variable value holds. There are, of course, many other uses of an expanded sales model. The example given was chosen for illustrative purposes only. Whenever such a model is used, the model considers all dependent adjustments that must be made in the firm's operations. The number of dependent relationships can often be so large, that a good estimate of profitability factors can *only* be obtained by some type of programming technique.

Production scheduling

Subproblem I, Section XIII, solved the problem of efficient product line scheduling. Once the quantities of the products to be manufactured were determined, the production scheduling problem was dependent only on the various production processes available, and the capacity limitations of productive resources. If a firm scheduled its manufacturing activities at given intervals (once a week, for example) then a much simpler model could be used for this production scheduling. In the "reduced-capacity" period chosen for our implementing study, the production scheduling problem is not large and probably does not require the use of any programming technique. However as economic activity increases, and the number of dependent relationships increase, the production scheduling problem

becomes more difficult and the value of a programming technique is greatly increased.

The dual variable associated with the capacity limitations of the simple production scheduling model are the marginal values of the productive resources. With proper interpretation, these dual variables can be extremely valuable in determining the value of increasing the capacity of some fixed production facility. Again, the estimation of these dual values, and the determination of the range over which these values hold, may be easily obtained for the simple case, but become much more difficult to obtain as the complexity of the problem increases.

A simple production model would also aid in the evaluation of new production alternatives as they are being developed. These new alternatives can not be evaluated in isolation, but must be considered in relation to all dependent changes which must be made in the firm's operations. The evaluation of the total effect upon the firm of any new production alternative may be extremely difficult without the aid of a programming technique which considers the many dependent relationships involved.

Multiplant firms

An expanded version of the model presented would be extremely valuable for a multiplant firm. In such a model,

separate processes must be included for manufacturing alternatives at each plant location. Selling processes would include the transportation costs from a particular plant to a particular distributor. A solution to an expanded model of this type would give proper consideration to both the efficiency of various manufacturing alternatives and the transportation costs involved. The associated dual variables, which represent the marginal values of constraint limitations, would likewise consider the many dependent relationships involved.

In a multiplant arrangement, the number of dependent relationships would be so great that it would be almost impossible to "guess" an optimal solution, or to "estimate" the marginal values of resources unless some type of programming technique was used. In summary, it can be said that the value of this type of programming increases with the complexity of the problem and the number of dependent relationships involved. The implicit assumption is, of course, that a reasonable model can be formulated.

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BIOGRAPHICAL SKETCH

Frank Sherman McLaughlin, Jr., was born on November 22, 1936 in Lake Wales, Florida. He attended public schools in Lake Wales and graduated from Lake Wales High School in 1954.

After graduating from high school, the author enrolled in Vanderbilt University. He was awarded the degree of Bachelor of Engineering *cum laude* in 1958. From 1958 until 1961 he served in the United States Navy aboard a destroyer in the Atlantic Fleet. On discharge from military service he entered the Graduate School at the University of Florida and received the MBA degree in 1962. He was then employed by Owens-Illinois as a chemical engineer to work in their pulp and paperboard mill in Jacksonville, Florida. He remained with this firm until 1965.

In 1965 the author received a Ford Foundation Fellowship in Economics and Business Administration and returned to the University of Florida to work toward the Doctor of Philosophy Degree. The author completed all requirements for the Ph.D. with a major in Economics and Business Administration in August, 1967. He has accepted

an appointment as an Assistant Professor of Business Administration at the University of Richmond starting in September, 1967.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Business Administration and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 12, 1967

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Dean, Graduate School

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